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Arab Republic of Egypt
Ministry of Education
Book Sector

MATHEMATICS

Student's Book

For Preparatory Year three — First Term

2015 - 2016

MATHEMATICS

Student's Book

For Preparatory Year three — First Term

* Learning mathematics enjoyably and usefully since it has real applications in your practical life.
* We have introduced you the subject in a simple way to help building mathematical knowledge and the patterns of sagacious thinking as well.
* Nous avons montré l'importance du rôle des mathématiques dans les autres domaines scientifiques.

دار مكة المكرمة للطباعة والنشر

3

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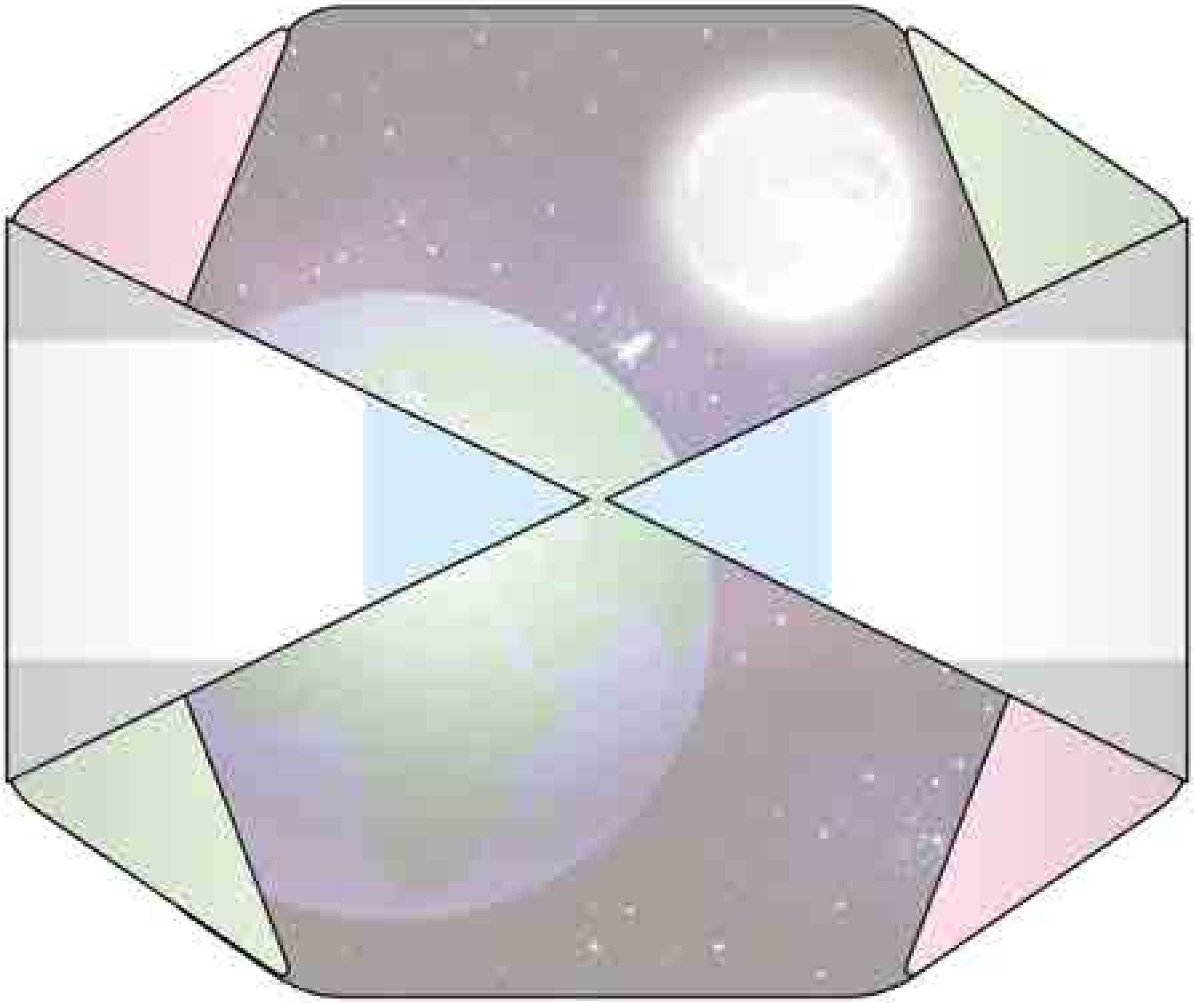

2015 - 2016

بسم الله الرحمن الرحيم

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Introduction

Dear students:

It is extremely great pleasure to introduce the mathematics book for third preparatory. We have been specially cautious to make learning mathematics enjoyable and useful since it has many practical applications in real life as well as in other subjects. This gives you a chance to be aware of the importance of learning mathematics, to determine its value and to appreciate mathematicians roles.

This book sheds new lights on the activities as a basic objective. Additionally, we have tried to introduce the subject simply and excitingly to help attaining mathematical knowledge as well as gaining patterns of positive thinking which pave your way to creativity.

This book has been divided into units, each unit contains lessons. Colors and pictures are effectively used to illustrate some mathematical concepts and the properties of figures. Lingual level of previous study has been taken into consideration.

Our great interest here is to help you get the information independently in order to improve your self-study skills.

Calculators and computer sets are used when needed. Exercises, practices, general exams, portfolios, unit test, general tests, and final term tests attached with model answers have been involved to help you review the curriculum completely.

Eventually, we hope getting the right track for the benefits of our students as well as for our dearest Egypt hoping bright future to our dearest students.

Authors

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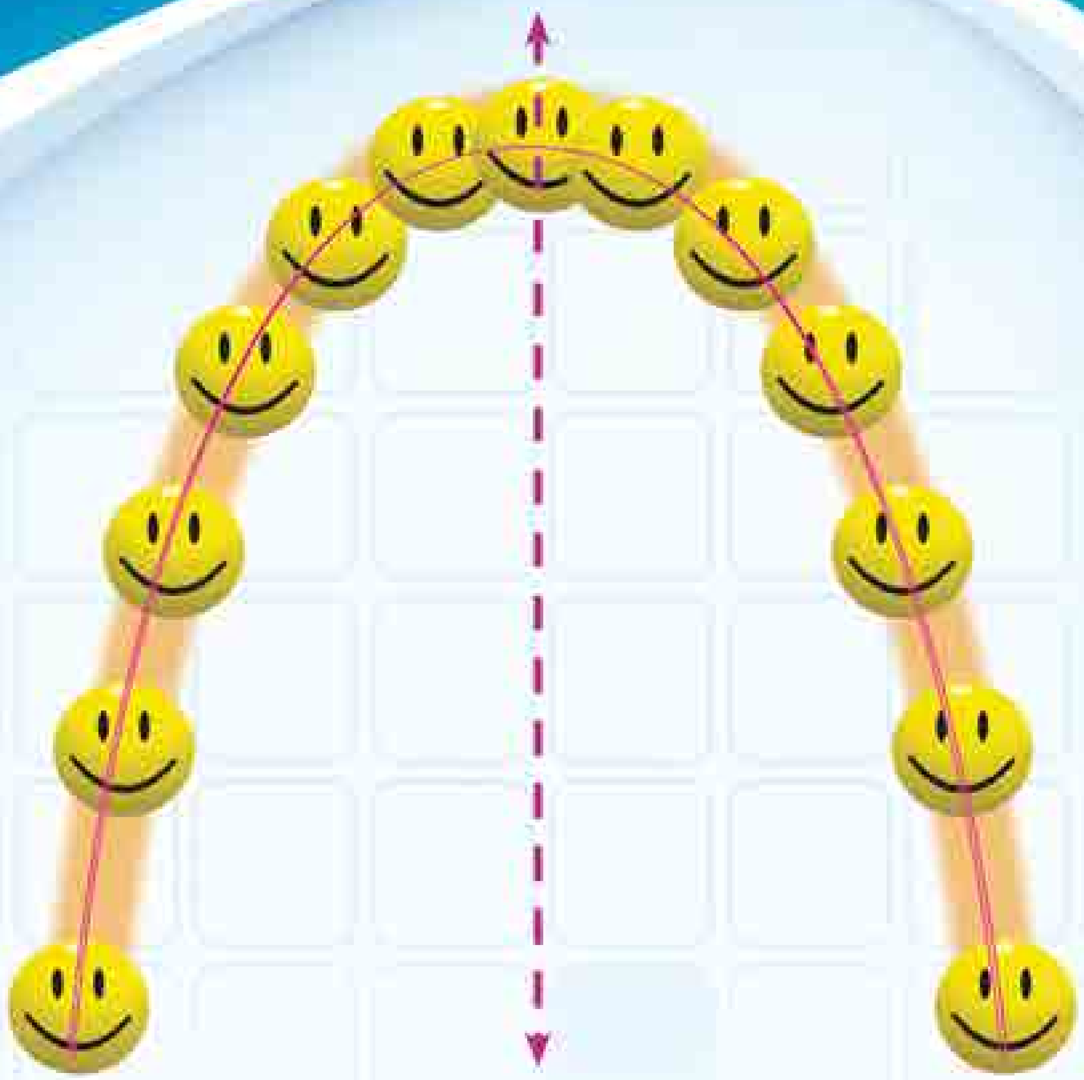
MATHEMATICAL NOTATION

N	The set of natural numbers	$\perp$	Perpendicular to
Z	The set of integers	$//$	Parallel to
Q	The set of rational numbers	$\overline{AB}$	Straight segment AB
Q`	The set of irrational numbers	$\overrightarrow{AB}$	Ray AB
R	The set of real number	$\overleftrightarrow{AB}$	Straight line AB
$\sqrt{A}$	The Square root of A	m ($\angle$ A)	Measure of angle A
$\sqrt[3]{A}$	The Cube root of A	m ($\overset{\frown}{AB}$)	Measure of arc AB
[a, b]	Closed interval	$\sim$	Similarity
]a, b[	Open interval	$>$	Grater than
[a, b[	Half-open interval	$\geqslant$	Grater than or equal to
]a,b]	Half-open interval	$<$	Less than
[a, ∞ [	Infinite interval	$\leqslant$	Less than or equal to
$\equiv$	Is congruent to	p(e)	Probability of occurring event (e)
n (A)	Number of elements of A	$\overline{x}$	Mean
s	Sample space	σ	Standard deviation
		$\sum$	Sum

Algebra

Unit 1

Relations and Functions



One of the players threw the ball so, it took the direction shown in the figure.

This figure represents one of the functions which you will study and is called " a quadratic function"

1 - 1 Cartesian product

What you'll learn

☆ Cartesian product of two non-empty sets.

Key terms

☆ Ordered pair.
☆ A cartesian product.
☆ An arrow diagram.
☆ A cartesian diagram.
☆ Relation.

Think and Discuss

You have previously studied relation between two variables x, y

1. Find a set of the ordered pairs which satisfy the relation: $y = 2x - 1$ when $x = 0$ and $x = 1$, $x = 2$
2. Represent these ordered pairs graphically in the coordinate plane.
3. Does the ordered pair (3, 5) equal the ordered pair (5, 3)? **(Use the graph).**

From the previous, we notice:

1. In each ordered pair (a, b), a is called the first projection, and b is called the second projection.
2. Each pair is represented by one and only one point in the coordinate plane.
3. *If* $a \neq b$ *then* $(a, b) \neq (b, a)$, Why?
4. $(a, b) \neq \{a, b\}$.
5. *If* $(a, b) = (x, y)$ *then* $a = x$, $b = y$

Example 1

Find x, y if: $(x - 2, 3) = (5, y + 1)$

Solution

$x - 2 = 5 \quad \therefore x = 7 \quad , \quad 3 = y + 1 \quad \therefore y = 2$

Drill

Find **a and b in each of the following:**

A. $(a, b) = (-5, 9)$
B. $(a - 2, b + 1) = (2, -3)$
C. $(6, b - 3) = (2 - a, -1)$
D. $(a - 7, 26) = (-2, b^3 - 1)$

Example 2

If $X = \{a , b\}$, $Y = \{-1 , 0 , 3\}$ then find: $X \times Y$, $Y \times X$, **What do you notice?**

Solution

To find the cartesian product of the set X and Y which is denoted by the symbol $X \times Y$, write the set of all the ordered pairs in which its, first projection is an element of X, and its second projection is an element belongs to Y , and it is written as:

$X \times Y = \{a , b\} \times \{-1 , 0 , 3\} = \{(a , -1) , (a , 0), (a , 3) , (b , -1), (b , 0), (b, 3)\}$

$Y \times X = \{-1, 0, 3\} \times \{a, b\} = \{(-1, a), (-1, b), (0, a), (0, b), (3, 0), (3, b)\}$

So: $X \times Y \neq Y \times X$

We can get $X \times Y$ and $Y \times X$ from the following tables

×		Second projection		
		-1	0	3
First Projection	a	(a, -1)	(a, 0)	(a, 3)
	b	(b, -1)	(b, 0)	(b, 3)

×		Second Projection	
		a	b
First projection	-1	(-1, a)	(-1, b)
	0	(0, a)	(0, b)
	3	(3, a)	(3,b)

Think:

1. When $X \times Y = Y \times X$?
2. Are the number of elements of $X \times Y$ = the number of elements of $Y \times X$?

We notice that :

1. *If* X and Y are two finite and non empty sets then :

   $X \times Y = \{(a , b) : a \in X , b \in Y\}$

2. $X \times Y \neq Y \times X$ *where* : $X \neq Y$

   $n(X \times Y) = n(Y \times X) = n(X) \times n(Y)$

   where n denotes the number of set elements .

3. *If* $(k, m) \in X \times Y$ *then* $k \in X$, $m \in Y$

4. *If* X is a non-empty set,

   then: $X \times X = \{(a , b) : a , b \in X\}$

   and written as X^2 and it is read as (X two).

Example 3

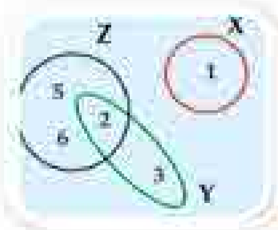
If X = {1}, Y = {2, 3}, Z = {2, 5, 6} represent the sets of X , Y , Z with venn diagram then find:

First : A $X \times Y$ B $Y \times Z$ C $X \times Z$ D Y^2

Second: $(X \times Y) \cup (Y \times Z)$ **Third:** $X \times (Y \cap Z)$

Fourth: $(X \times Y) \cap (X \times Z)$ **Fifth:** $(Z - Y) \times (X \cup Y)$

Solution



First :

A $X \times Y = \{1\} \times \{2, 3\} = \{(1, 2), (1, 3)\}$

B $Y \times Z = \{2, 3\} \times \{2, 5, 6\}$

$= \{(2, 2), (2, 5), (2, 6), (3, 2), (3, 5), (3, 6)\}$.

C $X \times Z = \{1\} \times \{2, 5, 6\} = \{(1, 2), (1, 5), (1, 6)\}$

D $Y^2 = Y \times Y = \{2, 3\} \times \{2, 3\}$

$= \{(2, 2), (2, 3), (3, 2), (3, 3)\}$

Second : $(X \times Y) \cup (Y \times Z) = \{(1, 2), (1, 3), (2, 2), (2, 5), (2, 6), (3, 2), (3, 5), (3, 6)\}$

Third : $X \times (Y \cap Z) = \{1\} \times \{2\} = \{(1, 2)\}$

Fourth : $(X \times Y) \cap (X \times Z) = \{(1, 2), (1, 3)\} \cap \{(1, 2), (1, 5), (1, 6)\} = \{(1, 2)\}$.

Fifth : $Z - Y = \{5, 6\}$ $\therefore (Z - Y) \times (X \cup Y) = \dots\dots\dots$ Complete

Drill

If X = {2 , - 1}, Y = {4 , 0} , Z = {4 , 5 , -2} Find

A $X \times Y$ B $Y \times Z$ C X^2

D $n(X \times Z)$ E $n(Y^2)$ F $n(Z^2)$

The representation of the cartesian product:

Example 4

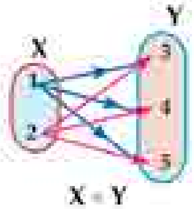
1 If X = {1, 2}, Y {3, 4, 5} ***Find:*** $X \times Y$, and represent it:

First: by the arrow diagram. **Second: by the cartesian diagram.**

Solution

$X \times Y = \{1, 2\} \times \{3, 4, 5\} = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$

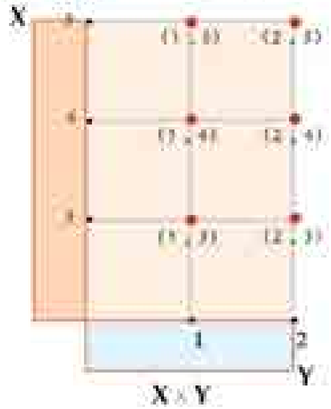
Where the cartesian product of $X \times Y$ is represented by an arrow diagram, or a grahpical net, as follows:



First: An arrow diagram

Draw an arrow from each element that represents the first projection (The elements of set of X) to each element that represents the second projection (The elements of set of Y)

i . e: The arrow diagram of the cartesian product represents each ordered pair by an arrow that starts from its first projection and ends at the second projection.



Second: Cartesian diagram (the perpendicular graphical net.

On a perpendicular graph net, the elements of set X is represented horizontally and the elements of set Y vertically. The intersection points of the horizontal and vertical lines represent the ordered pairs of the elements of the cartesian product $X \times Y$.

Example 5

If $X = \{3, 4, 8\}$ then find, $X \times X$ and represent it with an arrow diagram.

Solution

$X \times X = \{3, 4, 8\} \times \{3, 4, 8\}$

$= \{(3, 3), (3, 4), (3, 8), (4, 3), (4, 4), (4, 8), (8, 3), (8, 4), (8, 8)\}$.

Notice in the figure: the ordered pairs are represented by arrows, and the ordered pairs in which the first projection is equal to the second projection as: (3 , 3) , (4, 4), (8, 8) are represented by a buttonhole to show that the arrow comes from a point and ends in the same point.

Notice that: $n(X) = 3$, then $n(X \times X) = 3 \times 3 = 9$

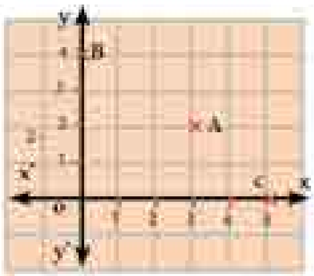
In this case, the cartesian product $X \times X$ can be represented graphically by 9 points where each point represents an ordered pair. But if X is an infinite set, then the number of elements of $X \times X$ is infinite.

Think: How can you represent the cartesian product of each of the following?

$N \times N$, $Z \times Z$, $Q \times Q$ and $R \times R$.

The cartesiem product of the infinite sets and its graphical representation:

First: **To represent the cartesian product of $N \times N = \{(x, y) : x \in N, y \in N\}$**



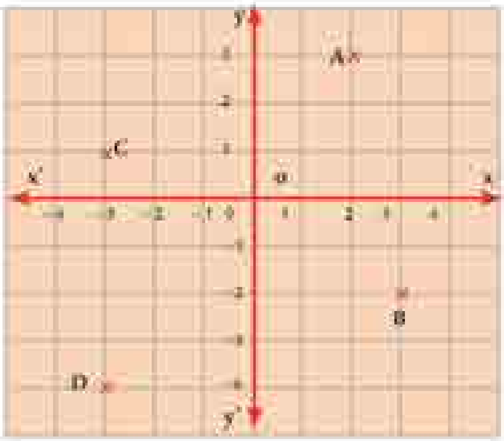
1. Draw two perpendicular straight lines, one of them is $\overleftrightarrow{xx`}$ horizontialy and the other $\overleftrightarrow{yy`}$ vertically and are intersected at point o.
2. Represent the natural numbers N on each of the horizontal and vertical straight lines starting with the origin point 0 which represents the number zero.
3. Draw vertical straight lines and horizontal straight lines from the points which represent the natural numbers, you will get the opposite figure, and thus, the points of intersection of the set of these straight lines are represented by the perpendicular graphical net of the cartesian product of $N \times N$.

Notice that: Each point of this net represents one the ordered pairs in the cartesian product of $N \times N$.

For Example: point A represents the ordered pair (3 , 2) and point B represents (0, 4).

Complete: point C represents the ordered pair (...... , ...), point O represents the ordered pair (...... , ...).

Second: **To represent the cartesien product of $Z \times Z = \{(x, y): x \in Z, y \in Z\}$.**



We represent the set of integers on each of the two horizontal and vertical straight lines where the point (O) represents the ordered pair (0, 0).
Thus, each point of the net points represents one of the pairs in the cartesian product $Z \times Z$
This net is known as the coordinat plane of $Z \times Z$.

Drill

Identify the ordered pairs which represented by the points A , B , C and D in the previous graphical net.

Third: **To represent the cartesian product $Q \times Q = \{(x, y) : x \in Q, y \in Q\}$**

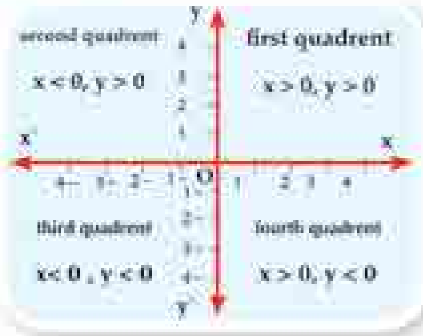
Draw a perpendicular graphical net and represent the set of rational numbers Q on the two horizontal and vertical straight lines, then identify the points: A $(3, \frac{5}{2})$, B $(-\frac{3}{2}, 4)$, C $(-3, -\frac{3}{2})$ and D $(\frac{5}{2}, -\frac{3}{2})$

Fourth: Representing the cartesian product RXR = {(x, y) : x ∈ R, y ∈ R}

the set of real numbers can be represented on each of the two horizontal and vertical straight lines, and point O represents the ordered pair (o , o).

The horizontal straight line $\overleftrightarrow{xx'}$ is called the x - axis, and the vertical straight line $\overleftrightarrow{yy'}$ is called the y - axis.

Then, the net is divided into four parts (quadrants) as in the opposite figure:



Example 6

Draw a perpendicular square net of the cartesian product R × R, then tell the quadrant or the axis where each of the following points is located:

A (3, 3), B (3, -2), C (-4, -2), D (-4, 3), E (0, - 3),F (2, 0)

Solution

A (3, 3) is located in the first quadrant.

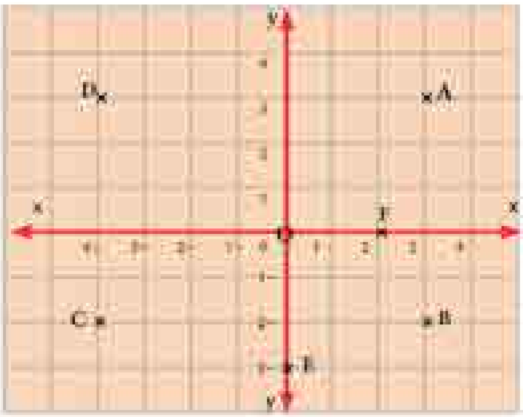
B (3, -2) is located in the fourth quadrant.

C (- 4, - 2) is the located in the third qudrant

D (- 4, 3) is the located in the second qudrant

E (0, - 3) is the locate on y - axis

F (2, 0) is the located on x - axis.



Drill

If X = { - 2 , 3} find the location which represents X × X.

Show which of the following points belongs to the cartesian product of X × X

A (1, 2), B (3, - 1), C (- 1, 4) and D (-2, 0)

Exercises 1-1

First : Complete the following:

1. If (a + 5, 3) = (8, b - 1) then a = ..., b = ...

2. If (x^5, y +1) = (32, $\sqrt[3]{27}$) then x = ..., y = ...

3. $(x - 1, 11) = (8, y + 3)$ then $\sqrt{x + 2y} = ...$
4. If $n(X^2) = 9$, , then $n(X) = ...$
5. If $X \times Y = \{(2, 6), (2, 9), (3, 6), (3, 9), (5, 6), (5, 9)\}$, then
   $X = ...$, $Y = ...$

Second: Choose the correct answer from the given answers::

1. If $n(X) = 3$, $n(X \times Y) = 12$ then $n(Y)$ equals: ....
   A. 4 B. 9 C. 15 D. 36
2. If $(3, 5) \in \{3, 6\} \times \{x, 8\}$ then $x =$ ....
   A. 8 B. 6 C. 5 D. 3
3. If the point $(5, b - 7)$ is located on the X - axis then $b =$
   A. 2 B. 5 C. 7 D. 12
4. If the point $(x - 4, 2 - x)$ where $x \in Z$ is located in the third quadrant, then x equals:
   A. 2 B. 3 C. 4 D. 6

Third:

1. If $X = \{2, 3\}$, $Y \{3, 4, 5\}$ then find:
   A. $X \times Y$ and represent it by an arrow diagram and and a cartesian diagram
   B. $n(X \times Y)$ C. $n(Y^2)$ D. $(X \times Y) \cap Y^2$
2. If $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$ then find:
   A. X, Y B. $Y \times X$ C. Y^2
3. If: $X = \{3, 4\}$, $Y = \{4, 5\}$, $Z = \{6, 5\}$ then find:
   A. $X \times (Y \cap Z)$ B. $(X - Y) \times Z$ C. $(X - Y) \times (Y - Z)$
4. Identify the following points on a perpendicular graphical net of the cartesian product $R \times R$:
   A (4, 5), B (6, -3), C (-2, 7), D (-1, 6), E (-4, -5), M (0, 6), K (9, 0)
   Then mention the quadrant that each point is located on the perpendicular graphichal net.
   Or the axis it belongs to.

1 - 2

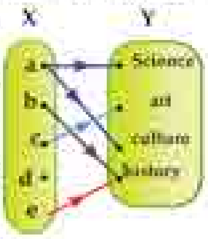
Relations

Think and Discuss

In the festival "Reading for All", five students represent the set of X = {a , b, c , d , e} went to the school library to read some books which are represented by the set Y = { science, literature, culture and history} student A read a book in science and a book in culture, student b read a book in histroy, student c read a literary book, pupil e read a book of the historical books, but student d didn't read any of these books.

1. Write the previous statements in the form of ordered pairs from X to Y.
2. Represent a set of the ordered pairs in the form of an arrow diagram.

Notice that: The expression " read" connects some of the elements of the set X with the elements of set Y, and it determines a relation between X and Y which is denoted by the symbol R. This relation can be represented by an arrow diagram - as shown in the opposite figure , where we draw an arrow beginning from the student and ending at the type of books he reads.



We can also express the relation from X to Y by the net of the following ordered pairs:

{(a, Science), (a, Culture), (b, Histrory), (c, Literature), (e, History)}.

This set of ordered pairs are called the relation R.

Think: Is R a subset from the cartesian product X × Z?

Example 1

If X = {-1, 1, 2}, Y = {2, 4, 6, 8}, and R is a relation from X to Y where a R b means: «b = 2 a + 4», for each $a \in X$, $b \in Y$

Write and represent R once in an arrow diagram and another by a cartesian diagram.

What you'll learn

- ☆ A relation of set of X to the set of Y.
- ☆ A relation from a set on it self.

Key terms

- ☆ Relation.

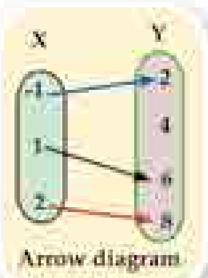
Solution

When: A = -1 $\quad \therefore B = 2 \times (-1) + 4 = 2$

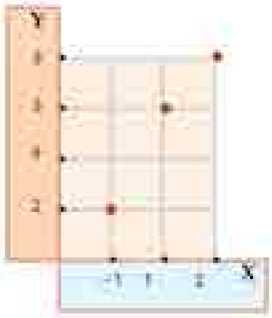
When: A = 1 $\quad \therefore B = 2 \times 1 + 4 = 6$

When: A = 2 $\quad \therefore B = 2 \times 2 + 4 = 8$

$\therefore R = \{(-1, 2), (1, 6), (2, 8)\}$



Arrow diagram



Graph digram

From the previous, we deduce that

1. The relation from X to Y where X, Y are two non-empty sets is a relation, connecting some or all the elements of X with some or all the elements of Y.
2. $X \times Y$ is the set of ordered pairs where the first projection in each ordered pair belongs to X and the second projection belongs to Y.
3. If R is a relation from X to Y , then $R \subset X \times Y$.

The relation from a set to itself

If R is a relation from a set X to X (itself) then R is called a relation on X and $R \subset X \times X$.

Example 2

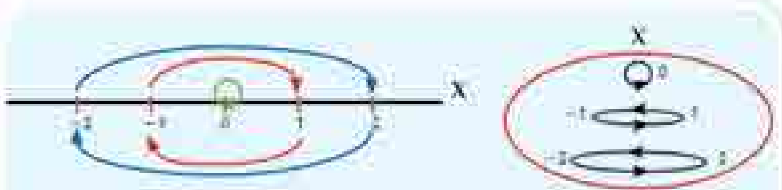
If X = {-2, -1, 0, 1, 2} and R is a given relation on X where a R b means :

«The number a is the additive inverse of the number b» for each of a , b $\in$ X

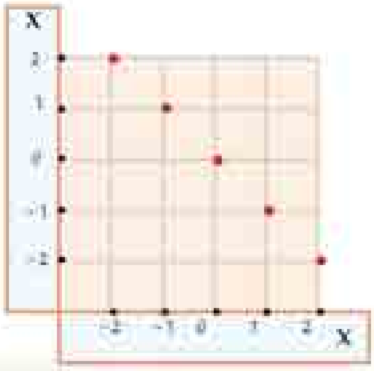
Write the relation R and represent it by an arrow diagram and also by, cartesian diagram.

Solution

R = { (-2, 2), (-1, 1), (0, 0), (1, -1), (2, -2)}



Arrow diagram



Cartesian Diagram

Drill

If X = {1, 2, 3}, Y = {12, 21, 47, 52}, and R is the relation from X to Y where a R b means : (a is a digit from the digits of b), for each $a \in X$, $b \in Y$

First: Write R and represent it by an arrow diagram and also, by a cartasian diagram.

Second: Show which of the following relations are correct and why?

1 R 52 2 R 21 3 R 47

Exercises 1-2

1. If X = {1, 2, 4, 6, 10}, and R is a relation on X, where a R b means **(a is a multiple of b)**, for each of a , $b \in X$. Write R and represent it by an arrow diagram and also, by a cartesian diagram.

2. If X = {2, 4, 5, 7}, Y = {4, 5, 6, 7, 9} and R is a relation from X to Y where a R b means ($a \leq b$) for each of $a \in X$ and $b \in Y$. Write R and represent it by an arrow diagram and also, by a cartasian diaram.

3. If X = {1, 2, 3}, Y = {1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$} and R is a relation from X to Y. where a R b means **«The number a is the multiplicative inverse of the number b»** for each of $a \in X$, $b \in Y$. Write R and represent it by an arrow diagrm and also, by a cartasian diagram.

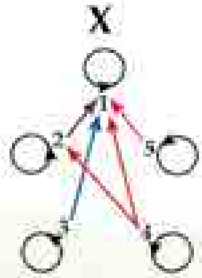
4. If X = {1, 3, 4, 5}, Y = {1, 2, 3, 4, 5, 6} and R is relation from X to Y where a R b means **«a + b = 7»** for each of $a \in X$, $b \in Y$. Write R and represent it by an arrow diagram and also by a cartesian diagram.

5. If X = {-1, 0, 1, 2, 3}, Y = {0, 1, 4, 6, 9} and R is a relation from X to Y where a R b means **«$a^2 = b$»** for each of $a \in X$, $b \in Y$. Write R and represent it by an arrow diagram and also by a cartesian diagram.

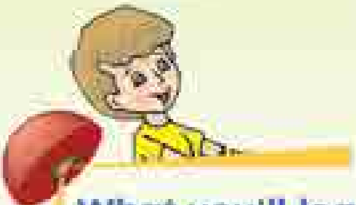
6. If X = {- 2, -1, 1, 2}, Y = {$\frac{1}{8}$, $\frac{1}{3}$, 1, 3, 8} and R is the relation from X to Y where a R b means «$a^3 = b$» for each of $a \in X$, $b \in Y$. Write R and represent it by an arrow diagram and also cartesian diagram.

7. If X = {2, 3, 4}, Y = {6, 8, 10, 11, 15} and R is a relation from X to Y. where a R b means **«a divides b»** for eah of $a \in X$, $b \in Y$ write the relation R .

8. **The opposite figure:**
Represents the arrow diagram of the given relation R on the set X = {1, 2 , 3, 4 , 5}. Write the relation R and represent it by a cartesian diagram.



1 - 3 Functions (Mapping)



What you'll learn

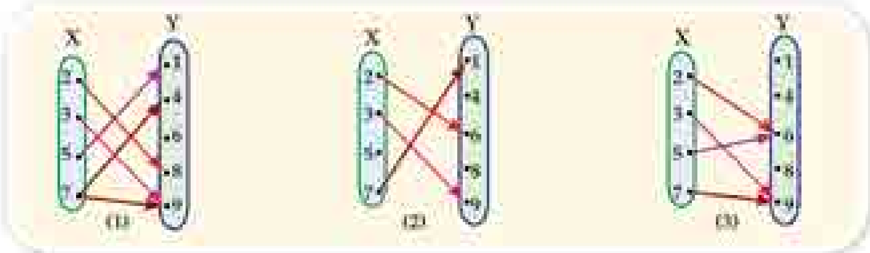
- ☆ Concept of the function.
- ☆ Symbolical expression of the function.

Key terms

- ☆ Functions.
- ☆ Domain
- ☆ Codomain
- ☆ Range

Think and Discuss

The following figures represent three relations from X to Y.



1. Write each relation and represent it by a cartesian diagram.
2. Which of these relations satifies the following condition: each element of X is connected to only one element of Y.

Definition:

A relation from X to Y is said to be a function if:

Each of the elements of X appears only once as a first projection in one of the ordered pairs of the relation.

The Symbolic representation of the function:

1. The function is denoted by one of the following symbols: f or m or Q or... and the function f from the set X to the set Y.

   is written mathimatecally as:

   $f : X \rightarrow Y$ and is read as: «f is a fuction from X to Y».

Notes:

1. If f is a function from X to itself, we say that f is a function on X.
2. If the ordered pair (x, y) belongs to the function, then the element y is called the image of the element x by the function f, and we express it by one of the following two forms:

   $f : x \mapsto y$ is read as : the function f maps x to y.

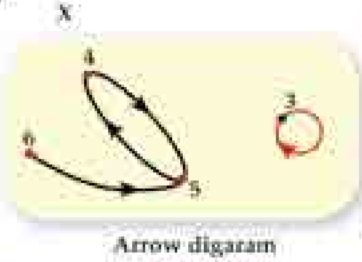
Or $f(x) = y$ it is read as: f is a function where $f(x) = y$

Example 1

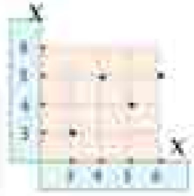
If f is a function on X where: X = {3, 4, 5, 6} and f (3) = 3, f (4) = 5, f (5) = 4, f (6) = 5.
Represent f by an arrow diagram and also, by a cartesian diagram.

Solution

f = { (3, 3), (4, 5), (5, 4), (6, 5)}



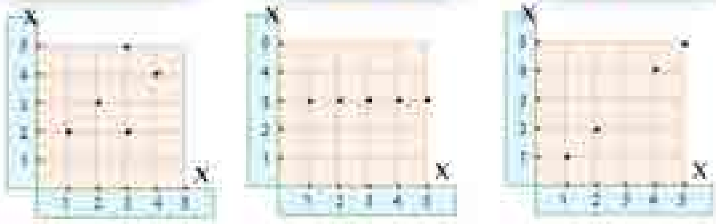
Arrow digaram


Cartesian diagram

Drill

1. If X = {1, 2, 3, 4} which of the following arrow diagrams represent a function on the set X?

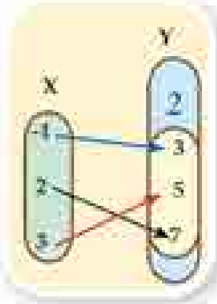
2. Which of the following cartesian diagrams represent a fuction from X to X.


(1) (2) (3)

Think: **Is every relation a function? Explain your answer and give examples.**

The Domain, the codomain and the range

If f is a function from X to Y.
i. e: f : X → Y , then
The set X is called the domain of the function f.
The set Y is called the codomain of the function f.
The set of images of the elements of the domain of X by the function f, is called the range of the function.



For example: If f : X → Y.
, X = { - 1, 2, 3}, Y = {2, 3, 5, 7}, f = { (-1, 3), (3, 5), (2, 7)} then:

1. The domain of the function f is the set X = { - 1, 2, 3}
2. The codomain of the function f is the set Y = {2, 3, 5, 7}
3. The range of the function f is the set of the images of the elements of X by the function f and equal to { 3, 5, 7}.

Note that: The range is a subset of the codomain of the function.

Example 2

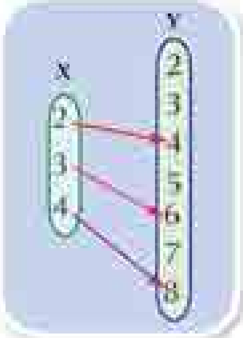
If X = {2, 3, 4}, Y = {y : y $\in$ N, 2 ≤ y < 9} where N is the set of natural numbers, and R is a relation from X to Y where a R b means: « $a = \frac{1}{2}b$ » for each of a $\in$ X, b $\in$ Y, write R and represent it by an arrow diagram show that R is a function from X to Y and find its range.

Solution

Y = {2, 3, 4, 5, 6, 7, 8} , R = {(2, 4), (3, 6), (4, 8)}

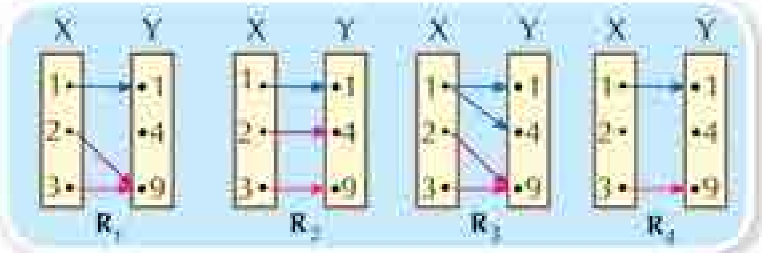
R is a function because every element of the X has only one arrow coming out to one element of Y.

The function range = {4, 6, 8}



Exercises 1-3

1. Which of the following relations represent a function from X to Y? If the relation represents a function, then find the function range?.



2. If X = {2, 5, 8}, Y = {10, 16, 24, 30} and R is a relation from X to Y where a R b means «a is a factor of b» for each a $\in$ X, b $\in$ Y. Write R and represent it by an arrow diagram and by cartesian diagram **Is R a function? and Why?**

3. If X = {0, 1, 4, 7} Y = {1, 3, 5, 6} and R is a relation from X to Y where a R b means «a + b < 8» for each a $\in$ X, b $\in$ Y, write R and represent it by an arrow diagram and also, by a cartesian diagram. Is R a function? and why?

4. If X ={1, 2 , 4 , 6 , 10} and R is a relation on X where a R b means: "a is twice b" for each of a , b $\in$ X. Write R, and represent it by an arrow diagram and also, by a cartesian diagram. Is R a function? and why?

5. If X = {1 , 2 , 3 , 6, 11} and R is a relation on X where a R b means: "a + 2 b = an odd number" for each of a, b $\in$ X, write R and represent it by an arrow diagram. Is R a function? and why?

1 - 4

Polynomial functions

Think and Discuss

In the functions $f: R \to R \ , \quad f_1(x) = 5$

$f: R \to R \ , \quad f_2(x) = 3x - 8$

$f: R \to R \ , \quad f_3(x) = 4x^2 - 5x + 8$

We notice that:

1. The domain and the codomain of the function is the set of the real numbers R.
2. The rule of function (image of x) is a term or an algebraic expression.
3. What the power of the variable x in the previous functions?

Definition

The function $f: R \to R$ where:

$f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$ where $a_0, a_1, a_2, \dots a_n \in R$
$n \in N$, $a_n \neq 0$, is called a polynomial of degree n.

And thus: the degree of the polynomial is the highest power of the variable in the function rule.

Drill

1. Which of the following functions represents polynomial:

   A) $f_1(x) = x^3 + x^2 + 3$ B) $F_2(x) = x^3 + \frac{1}{x} + 7$

   C) $f_3(x) = x^2 + \sqrt{x} + 8$ D) $F_4(x) = x\left(x + \frac{1}{x} - 2\right)$

2. If $f: R \to R$ then mention the degree of the function in the following:

   A) $f(x) = 3 - 2x$ B) $f(x) = x^2 - (x^2 - 3)$

   C) $f(x) = x(x - 2x^2)$ D) $f(x) = x^2(x - 3)^2$

What you'll learn

- ☆ The linear function and its graphical representation.

Key terms

- ☆ Polynomial function.
- ☆ Linear function.
- ☆ quadratic Function
- ☆ The graphical representation of function.

Example 1

If $f(x) = x^2 - x + 3$ then find: $f(-2)$, $f(0)$, $f(\sqrt{3})$

Solution

$\because f(x) = x^2 - x + 3 \quad \therefore f(-2) = (-2)^2 - (-2) + 3 = 4 + 2 + 3 = 9$

$f(0) = 3$, $\quad f(\sqrt{3}) = (\sqrt{3})^2 - \sqrt{3} + 3 = 6 - \sqrt{3}$

Drill

If $f(x) = x^2 - 3x$, $\quad g(x) = x - 3$

A Find $f(\sqrt{2}) + 3\,g(\sqrt{2})$

B Prove that $f(3) = g(3) = 0$

Linear function

Definition

The function $f: R \to R$ where $f(x) = ax + b$, $a, b \in R$, $a \neq 0$ this function is called a linear function or a function of the first degree.

The graphical representation of the linear fuction:

Example 2

Represent graphically the function $f: R \to R$, $f(x) = 2x - 3$

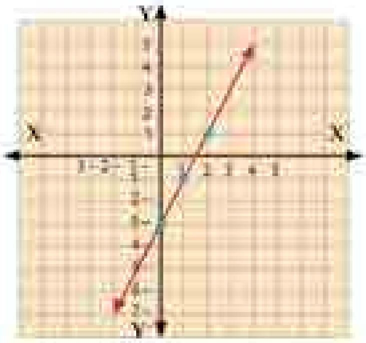
Solution

$\because f(x) = 2x - 3$

$\therefore f(0) = 0 - 3 = -3$, $f(1) = 2 - 3 = -1$, $f(2) = 4 - 3 = 1$

You can put these ordered pairs in a table as the following:

x	0	1	2
y = f(x)	-3	-1	1



The ordered pairs of the cartesian product of $R \times R$ is represented on the square net.

Remarks:

1. It is enough to find two ordered pairs belonging to the function, it is prefered to find third ordered pairs to check the graph.
2. If $f : R \to R$, $f(x) = a\,x$, where $a \neq 0$ then it represents graphically by a straight line passing through the origin (0, 0)

Drill

Reprsent graphically each of the following functions:

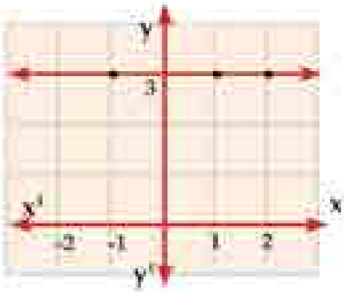
1. $f: f(x) = x + 2$
2. $g : g(x) = 3\,x$
3. $\ell : \ell(x) = -2\,x$

Special case: If $f : R \to R$, $f(x) = b$ where $b \in R$

then f is called a constant function.

For example: $f(x) = 3$

and it is written as $y = 3$



x	-1	1	2
y = f(x)	3	3	3

It is represented by a straight line paralled to the x-axis.

Drill

Represent the following functions graphically:

1. $f(x) = 5$
2. $f(x) = -4$
3. $f(x) = 0$
4. $f(x) = 2\frac{1}{2}$

The quadratic function

The function $f : R \to R$ where $f(x) = a\,x^2 + b\,x + c$, a, b, c are real numbers, $a \neq 0$ is called a quadratic function and it is a function of second degree.

The graphical representation of the quadratic function.

Example 3

Represent graphically the quadratic function f, where $f(x) = x^2$, $x \in R$ consider $x \in [-3, 3]$

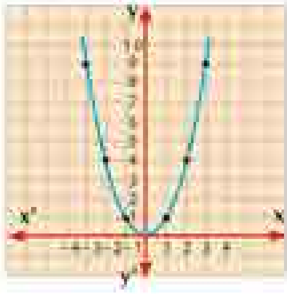
Solution

Identify some of the ordered pairs (x , f (x)) which belong to the function f where $x \in R$ and that the interval is [- 3 , 3] gives some possible values the variable x.

$f(-3) = 9$,$f(-2) = 4$, $f(-1) = 1$, $f(0) = 0$, $f(1) = 1$, $f(2) = 4$, $f(3) = 9$

Put these ordered pairs in a table as follows:

x	3	2	1	0	-1	-2	-3
y = f(x)	9	4	1	0	1	4	9



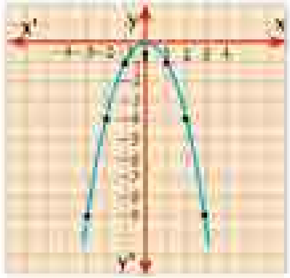
Identify in the cartesian plane the points which represent these ordered pairs, then draw a curve passing through these points.

Notice that:

1. The curve of the function f is symmetrical about the y-axis and the equation of the symmetrical axis is $x = 0$
2. The coordinate of the vertex of the curve is (0, 0), and the minimum value of the function = 0

Example 4

Represent graphically the quadratic function f where:
$f(x) = -x^2, x \in R$ where $x \in [-3, 3]$



Solution

Repeat the previous solution steps:

x	3	2	1	0	-1	-2	-3
y = f(x)	-9	-4	-1	0	-1	-4	-9

From the previous drawing, we notice:

1. The curve of the function f is symmetrical about the y-axis, thus, the equation of the symmetrical axis is $x = 0$
2. The coordinate of the vertex of the curve is (0, 0) and the maximum value of the function = 0

Exercises 1-4

First: Complete the following:

1. The linear function given by the rule $y = 2x - 1$ is represented graphically by a straight line intersecting the y-axis at the point ....
2. The linear function given by the rule $y = 3x + 6$ is represented graphically by a straight line intersecting the x - axis at the point ....
3. If the point (a, 3) is located on the straight line which represents the function $f: R \to R$ where $f(x) = 4x - 5$ then a equals ....

Second: 1. If $f: R \to R$, mention the degree of f then find $f(-2)$, $f(0)$, $f(\frac{1}{2})$ when:

A. $f(x) = 3$ B. $f(x) = 3 - 2x$ C. $f(x) = x^2 - 4$

2. Represent grapically the following linear functions and find the points of intersection of the straight line by the two coordinate axes:

A. $f(x) = 2x$ B. $f(x) = -\frac{1}{2}x$ C. $f(x) = 2x + 1$

D. $f(x) = 2 - x$ E. $f(x) = 3x - 1$ F. $f(x) = -2x + 3$

3. Represent graphically each of the following functions and from the drawing deduce the coordinate of the vertex of the curve, and the equation of the symmetry axis and the minimum and the maximum value of the function.

A. $f(x) = x^2 - 2$ where $x \in [-3, 3]$ B. $f(x) = (x - 2)^2$ where $x \in [-1, 5]$

C. $f(x) = x^2 + 2x + 1$ where $x \in [-4, 2]$ D. $f(x) = 2 - x^2$ where $x \in [-3, 3]$

Connecting with technology

Using computer programs:

There are many free computer programs to draw the curves and solve the equations. It is available on the worldwide web, such as the free program (GeoGebra) its url is: http://www.geogebra.org, the program supports Arabic language.

By using the program, represent graphically each of the following functions:

1. $f(x) = 2x + 1$ 2. $f(x) = 5 - 3x$
3. $f(x) = x^2 - 3x + 2$ 4. $f(x) = 4 - 3x - x^2$

Activity

1. A Pavement company gets paid 100.000 pounds (fixed fee) then 30 pounds for each meter. If X (the length of the paved road in meters) and **Y** is (the total cost that the company recieves).

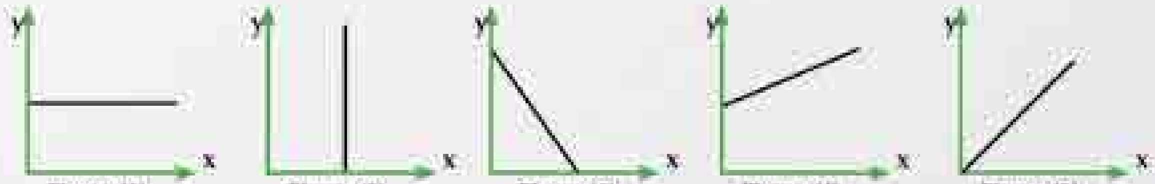


Figure (1)   Figure (2)   Figure (3)   Figure (4)   Figure (5)

First: The figure that represents the relation between x and y is the figure number ....

Second: Which of the following relations represents the previous infmation:

A. $y = 30x$ B. $y = 30x + 100000$ C. $y = 100000x + 30$ D. $y = 3000000x$

Third: Write an essay about the great efforts of our country to improve and pave the roads to be faster and safer. Discuss what you should follow such obeying traffic laws and keeping the roads clean and safe.

Unit Test

1. If $X = \{0, 1, 4, 7\}$, $Y = \{1, 3, 5, 6\}$, R is a relation from X to Y where a R b means: «$a + b < 6$» for each of $a \in X$, $b \in Y$, write R and represent it by an arrow diagram and by cartesian diagram. Is R a function? Tell the reason.

2. Represent graphically each of the following functions:

   A. $f(x) = 3x - 1$
   B. $f(x) = -2x$
   C. $f(x) = x^2 - 3$ where $x \in [-3, 3]$
   D. $f(x) = 1 - 3x + x^2$ where $x \in [-1, 4]$

3. While Karim was reading a book, he found that after 3 hours 50 pages remained, and after 6 hours, 20 pages remained. If the relation between time (t) and the number of pages (b) is a linear relation.

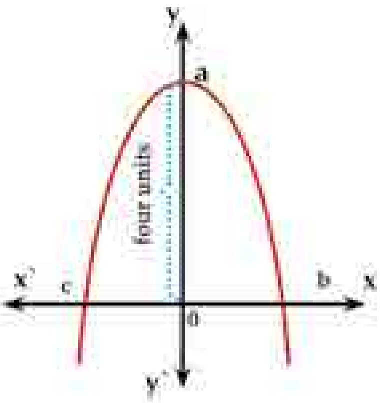
   A. Represent graphically the relation between t and b then find the algebraic relation between the two variables.
   B. What is the time that should be taken to finish the book?
   C. What are the number of pages remaining when Karim began to read?

4. **The opposite figure :** Represents the curve of the function f, where:

   $f(x) = m - x^2$, If $a o = 4$ units

   Find:

   A. The value of m.
   B. The coordinates of b and c
   C. The area of the triangle with vertices a , b and C.



Algebra

Unit 2: Ratio, proportion, Direct Variation and Inverse Variation

Do you Know ?

The weight of a human body on the surface of the moon equals $\frac{1}{6}$ of the weight on the surface of Earth.

Imagine you are going to a trip on the moon: What will your weight be?

2 - 1 Ratio

What you'll learn

- ☆ Ratio.
- ☆ Properties of ratio.

Key Terms

- ☆ Antecedent.
- ☆ Consequent.
- ☆ The two terms of the ratio.

Think and Discuss

We have learned in the previous phases the subject of ratio and that ratio is: a comparison between two quantaties.

for example: If there are 4 boys and 3 girls so the ratio between the number of boys to the number of girls can be written as 4 to 3 or $\frac{4}{3}$ Generally, if a and b are two real numbers

Then, the ratio between the two numbers a and b

Can be written as a to b or a:b or $\frac{a}{b}$.

a will be called an antecedant and b is consequent and a and b together are the two terms of ratio.

Complete and answer the questions:

1. Is the ratio changed if each of its two terms is multiplied in a fixed amount not equalling to zero?

   $\frac{3}{5} \overset{?}{=} \frac{3\times ...}{5\times ...}$

2. Is the ratio changed if you add a real number to each of its two terms?

   $\frac{2}{3} \overset{?}{=} \frac{2+ ...}{3+ ...}$

3. If $\frac{a}{b} = \frac{3}{5}$, Is a = 3, b = 5 for the values of a and b?

Example

Find the number which if added to the two terms of ratio 7 : 11 it will be 2 : 3

Solution

Consider the number is x.

$\therefore \frac{x+7}{x+11} = \frac{2}{3}$ $\therefore 3(x+7) = 2(x+11)$

$\therefore 3x + 21 = 2x + 22$ $\therefore 3x - 2x = 22 - 21$

$\therefore x = 1$

Drill

Find the positive number which if we add its square to each of the two terms of ratio 5 : 11 it becomes 3 : 5

Exercises (2-1)

1. Two integer numbers, the ratio between them is 3 : 7 and if subtracted 5 from each term, the ratio between each of them becomes 1 : 3. Find the two numbers?

2. Two integer numbers, the ratio between them is 2:3, if you add to the first 7 and substract from the second 12, the ratio between them becomes 5 : 3, find the two numbers?

3. ***Find*** the number that if subtracted thrice of it from each of the two terms of ratio $\frac{49}{69}$ the ratio becomes $\frac{2}{3}$.

4. ***Find*** the number which if its square is added to each of the two terms of the ratio 7:11 it becomes 4:5.

2 - 2 Proportion

What you'll learn

- ☆ Proportion
- ☆ Properties of proportion
- ☆ Continued properties

Key Terms

- ☆ Proportion
- ☆ First proportional
- ☆ Second proportional
- ☆ Third proportional
- ☆ Fourth proportional
- ☆ Extremes
- ☆ Means

If $\frac{a}{b} = \frac{c}{d}$ then it's said that a, b, c and d are in proportion.

If a, b, c and d are in proportion, **then** $\frac{a}{b} = \frac{c}{d}$

> **Definition:**
>
> **The proportion is the equality of two ratios or more.**

In ratio $\frac{a}{b} = \frac{c}{d}$

So, ***a*** is called the **first proportional**, ***b*** is called the **second proportional**, ***c*** is called the **third proportional**, and ***d*** is called the **fourth proportional**.

a and d are called extremes, b and c are called means

The proporties of proportion

first: if $\frac{a}{b} = \frac{c}{d}$ ***then:***

1. $a = m\,c$, $b = m\,d$ **where** $m \in R^*$
2. $a\,d = b\,c$ **(product of the extremes equals product of the means)**
3. $\frac{a}{c} = \frac{b}{d}$

Check the previous properties by giving numerical examples of your own

Second: If: $ad = bc$ ***then :*** $\frac{a}{b} = \frac{c}{d}$

$\frac{a}{c} = \frac{b}{d}$

Check the proporties in the following numeric example:

You know that: $4 \times 8 = 2 \times 16$

then: $\frac{4}{2} = \frac{\dots}{\dots}$, $\frac{4}{16} = \frac{\dots}{\dots}$

Example 1

If $\frac{x}{y} = \frac{2}{3}$ find the value of the ratio: $\frac{3x+2y}{6y-x}$

Solution

Consider $x = 2m$, $y = 3m$ (where m constant ≠ zero)

$$\therefore \frac{3x+2y}{6y-x} = \frac{3\times 2m + 2\times 3m}{6\times 3m - 2m} = \frac{12m}{16m} = \frac{3}{4}$$

Another Solution

Divide the numerator and denominator on y, then substitute for the value of $\frac{x}{y}$

$$\therefore \text{The expression} = \frac{3\times \frac{x}{y} + 2}{6 - \frac{x}{y}} = \frac{3\times \frac{2}{3} + 2}{6 - \frac{2}{3}} \longrightarrow \textit{Complete} = \frac{\ldots}{\ldots} = \frac{\ldots}{\ldots}$$

Example 2

Find the fourth proportional for the numbers 4, 12, 16

Solution

Consider the fourth proportional to be x

$$\frac{4}{12} = \frac{16}{x}$$

$\therefore 4 \times x = 12 \times 16$ ***[product of the extremes = product of the means]***

$\therefore x = \frac{12\times 16}{4} = 48$ $\therefore$ The fourth proportional $= 48$

Example 3

Find the number that if added to the numbers 3, 5, 8 and 12 it becomes proportional .

Solution

Consider the number is x i.e. $3 + x$, $5 + x$, $8 + x$, $12 + x$ **are in proportional**

$\therefore \frac{3+x}{5+x} = \frac{8+x}{12+x}$ $\therefore (5 + x)(8 + x) = (3 + x)(12 + x)$

$\therefore 40 + 13x + x^2 = 36 + 15x + x^2$ $\therefore 15x - 13x = 40 - 36$

$\therefore 2x = 4$ $\therefore x = 2$

Drill

1. **A** Find the second proportional of the numbers 2, ... , 4, 6

   B Find the third proportional of the numbers 8, 6, ... , 12

2. If $\frac{a}{b} = \frac{3}{5}$ find the value of $7a + 9b : 4a + 2b$

Third If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots\ldots,\ m_1, m_2, m_3, \ldots \in R^*$

then: $\frac{a\,m_1 + c\,m_2 + e\,m_3 + \ldots}{b\,m_1 + d\,m_2 + f\,m_3 + \ldots}$ = one of the ratios

For example: If: $\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$ multiply the first two terms of the first ratio by 2, multiply the two terms of the second ratio by -5 and multiplying the two terms of the third ratio by 3, then

$\frac{2a - 5b + 3c}{2 \times 2 - 3 \times 5 + 3 \times 4}$ = one of these ratios

i.e.: $2a - 5b + 3c$ = one of these ratios

Example 4

If: a, b, c and d are proportional quantities, ***then prove that :*** $\frac{3a - 2c}{5a + 3c} = \frac{3b - 2d}{5b + 3d}$

Solution

$\because$ If a , b , c and d are proprotional quantities $\qquad \therefore \frac{a}{b} = \frac{c}{d}$

Multiply the first two means by five and the second means by 3, then the sum of antecedents and the sum of consequents = one of these ratios .

$\therefore \frac{5a + 3c}{5b + 3d}$ = one of these ratios $\qquad$ (1)

Multiply the two terms of ratio by 3 and the second by -2 then the sum of antecendents : the sum of consequents = one of these ratios .

$\therefore \frac{3a - 2c}{3b - 2d}$ = one of these ratios $\qquad$ (2)

from (1), (2) $\therefore \frac{5a + 3c}{5b + 3d} = \frac{3a - 2c}{3b - 2d}$ $\qquad \therefore \frac{3a - 2c}{5a + 3c} = \frac{3b - 2d}{5b + 3d}$ $\qquad$ **(Q.E.D)**

Another Solution

Consider $\frac{a}{b} = \frac{c}{d} = m$ where m is a constant expression

$a = b\,m$, $c = d\,m$ and substitute in both sides.

Drill

If $\frac{a}{b} = \frac{c}{d}$ ***prove that :***

First: $\frac{a+b}{b} = \frac{c+d}{d}$ **Second:** $= \frac{a-b}{b} = \frac{c-d}{d}$

Hint Consider $\frac{a}{b} = \frac{c}{d} = m$ where m is a constant expression ≠ zero and complete or in any other way.

Continued proportional

2, 6, 18 are three numbers. Compare between the proportions $\frac{2}{6}$, $\frac{6}{18}$

1. Is there a relation between $(6)^2$ and the product of 2×18?
2. If you replace the number 6 with (-6) is there a relation between $(-6)^2$ and the product of 2×18?

Definition:

The quantities a, b and c are said to be in continued proportional if:
$\frac{a}{b} = \frac{b}{c}$ **a is called the first proportional , b is called the middle proportional, and c is called the third proportional, where : $b^2 = ac$ or $b = \pm\sqrt{ac}$**

Example 5

Find the middle proportional between 3, 27

Solution

The middle proportional $= \pm\sqrt{3 \times 27} = \pm 9$

Example 6

If b is a middle proportional between a and c, prove that : $\frac{a^2+b^2}{b^2+c^2} = \frac{a}{c}$

Solution

b is middle proportional between a and c i.e. a, b, c in continued proportional

Consider $\frac{a}{b} = \frac{b}{c} = m$ $\therefore b = c\,m$

$a = b\,m = c\,m \times m = c\,m^2$

L.H.S $= \frac{a^2+b^2}{b^2+c^2} = \frac{c^2 m^4 + c^2 m^2}{c^2 m^2 + c^2}$

$= \frac{c^2 m^2 (m^2+1)}{c^2 (m^2+1)} = m^2$ **(1)**

R.H.S $= \frac{a}{c} = \frac{c\,m^2}{c} = m^2$ **(2)**

From **(1)**, **(2)** we get $\frac{a^2+b^2}{b^2+c^2} = \frac{a}{c}$

Another Solution

Consider : $\frac{a}{b} = \frac{b}{c} = m$ $\therefore \frac{a^2}{b^2} = \frac{b^2}{c^2} = m^2$

From the first ratio and the second ratio $m^2 = \frac{a^2+b^2}{b^2+c^2}$ = L.H.S

$m^2 = \frac{a}{b} \times \frac{b}{c} = \frac{a}{c}$ = R.H.S

From **(1)**, **(2)** $\therefore \frac{a^2+b^2}{b^2+c^2} = \frac{a}{c}$

Drill

If a , b , c and d are in continued proportional . Prove that : $\frac{a-2b}{b-2c} = \frac{3b+4c}{3c+4d}$

Hint Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

then $c = dm$, $b = dm^2$, $a = dm^3$ complete

Exercises (2-2)

1. **If:** $\frac{y}{x-z} = \frac{x}{y} = \frac{x+y}{z}$ **Prove that each ratio is equal to** 2 (unless: $x + y = 0$). ***then Find*** $x : y : z$

2. **If** $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3x}$ ***then find*** the value of x.

3. **If $a : b : c = 5 : 7 : 3$ and $a + b = 27.6$ *then find* the value of a, b and c**

4. **If x, y, z and ℓ are proportional quantities then prove that :**

   A) $\left(\frac{x+y}{z+\ell}\right)^2 = \frac{2x^2 - 3y^2}{2z^2 - 3\ell^2}$

   B) $\sqrt[3]{\frac{5x^3 - 3z^3}{5y^3 - 3\ell^3}} = \frac{x+z}{y+\ell}$

5. **If** $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ ***then prove that:***

   A) $\frac{2y - z}{3x - 2y + z} = \frac{1}{2}$

   B) $\sqrt{3x^2 + 3y^2 + z^2} = 2x + y$

6. If a, b, c and d are four real proportional quantities.

   then prove that:

   A) $\frac{ac}{bd} = \left(\frac{a-c}{b-d}\right)^2$

   B) $\sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \frac{a+c}{b+d}$

7. If b is the middle proportional between a and c, then prove that:

   A) $\frac{a+b+c}{a^{-1} + b^{-1} + c^{-1}} = b^2$

   B) $\frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{c}{a} = \frac{c^2}{b^2}$

8. If a , b, c and are d in continued proportional, ***then prove that:***

   A) $\frac{ab - cd}{b^2 - c^2} = \frac{a+c}{b}$

   B) $\frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{b}{d}$

   C) $\frac{a}{b+d} = \frac{c^3}{c^2 d + d^3}$

   D) $\frac{c^2 - d^2}{a - c} = \frac{bd}{a}$

9. If : 5a, 6b, 7c and 8d are positive quantities in continued proportional ,

   Prove that : $\sqrt[3]{\frac{5a}{8d}} = \sqrt{\frac{5a + 6b}{7c + 8d}}$

2 - 3

Direct Variation and Inverse Variation

What you'll learn

- ☆ Direct variation
- ☆ Inverse variation
- ☆ Difference between direct variation and inverse variation.

Key Terms

- ☆ Variation
- ☆ Direct variation
- ☆ Inverse variation

First: Direct variation

Think and Discuss (1)

A car moves at a uniform velocity (V) 15 m/sec. If the covered distance (d) in meter in a time (t) per second to give the relation: $d = v\,t$.

t	1	2	3	4
d	15	30	45	60

A Represent the relation between d and t graphically.

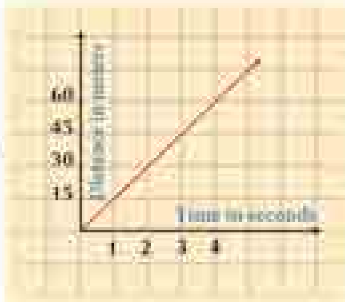
B Does the graphical representation pass through the origin point (0, 0)?

C Find $\frac{d}{t}$ in each case, what do you notice?

We notice from the above :

$\frac{d}{t}$ equals a constant expression which is 15

i.e.: d = 15 n and is said to be directly due to n and written symbolically $d \propto n$.



Definition:

y is said to be varies directly with x and is written as $y \propto x$ and written $y = m\,x$ (where m constant $\neq 0$). If the variable x takes the two values x_1, x_2 and the variable y takes the two variables y_1, y_2 respectively , then: $\frac{y_1}{y_2} = \frac{x_1}{x_2}$

From the previous, we conclude:

1. The previous relation is a linear relation between x and y and the two variables x and y, and is represented by a straight line passing through the origin point.
2. If $y \propto x$ then $y = m\,x$
   and it $y = m\,x$ then $y \propto x$.

Example 1

If $y \propto x$; $y = 14$ when $x = 42$, ***then find***

first: the relation between x and y **second:** find the value of y when $x = 60$

Solution

First: $\because y \propto x$ $\therefore y = m\,x$ (where m constant $\neq 0$)

subtitute for the values of x and y in the relation

$\therefore 14 = 42 \times m$ $\therefore m = \frac{14}{42} = \frac{1}{3}$ $\therefore$ the relation is: $y = \frac{1}{3}\,x$

Second: when $x = 60$ $\therefore y = \frac{1}{3} \times 60 = 20$

notice : You can find the relation $\frac{y_1}{y_2} = \frac{x_1}{x_2}$ to find the value of y in the second requirement

Second: Inverse variation

If the area of the rectangle m and one of both dimensions x and the other dimension y, then:

A. ***Write*** the relation between m, x and y.

B. If the area of the rectangle is constant and equal to 30 cm² ***complete*** the following table:

x	3	5	6	10
y	....	....	....	....

C. ***Find*** x y in each case . What do you notice?

From the previous , we notice that:

$x\,y = 30$ *i.e.:* $y = \frac{30}{x}$ *i.e.* y inversely changes with Y and Written symbolically $y \propto \frac{1}{x}$

Similarly: $x = \frac{30}{y}$ *i.e.:* x inverserly changes with Y and Written symbolically $x \propto \frac{1}{y}$

Definition:

y is said to be changed inversely with x and written $y \propto \frac{1}{x}$ if $x\,y = m$ (where m constant $\neq 0$)

and if the variable x takes the two values x_1, x_2 accordingly, the variable y takes the two values y_1, y_2 respectively: $\frac{y_1}{y_2} = \frac{x_2}{x_1}$

From the previous, we conclude that :

1. The previous relation is not a linear relation between the two variables x and y and is not represented by a straight line .

2. If y inversely changes with x ***then:*** $y = \frac{m}{x}$ (where m constant $\neq 0$)
   and if $y = \frac{m}{x}$ then $y \propto \frac{1}{x}$.

Example 2

If $y \propto \frac{1}{x}$ and $y = 3$ when $x = 2$

first: ***find*** the relation between x and y. **second:** ***find*** the value of y when $x = 1.5$.

Solution

$\because y \propto \frac{1}{x}$ $\therefore y = \frac{m}{x}$ (where m constant $\neq 0$)

substitute for the two values of x and y in the relation

$\therefore 3 = \frac{m}{2}$ $\therefore m = 2 \times 3 = 6$

$\therefore$ the relation is : $y = \frac{6}{x}$

when $x = 1.5$ $\therefore y = \frac{6}{1.5} = 4$

Note: you can find the value of y from the relation $\frac{y_1}{y_2} = \frac{x_2}{x_1}$

Drill

Show **which of the following tables represents the direct variation and which represents the inverse variation and which does not represent the direct variation or inverse variation while mentioning the reason in each case:**

x	y
3	20
5	12
4	15
6	10

x	y
2	9
4	18
12	54
16	72

x	y
5	9
10	18
15	27
25	45

x	y
3	6
-2	-9
-18	1
9	-2

Example 3

Connecting with Physics : If the relation between velocity (v) in (m/sec) and time t (sec) is $v = 9.8\,t$

First: ***determine*** the kind of variation between v and t .

Second: **A** ***Find*** the values of v when $t = 2$ seconds , $t = 4$ seconds

B ***Find*** the value of t when $v = 24.5$ m/sec

Solution

First: $\because v = \text{constant} \times t$ *i.e.* $v \propto t$ *i.e.* v directly changes with **t**.

Second: **A** when $t = 2$ then $v = 9.8 \times 2 = 19.6$ m/s

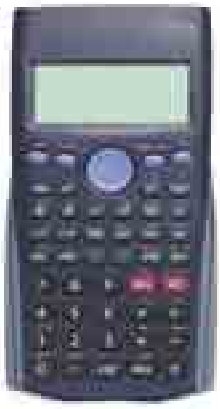
when $t = 4$ then $v = 9.8 \times 4 = 39.2$ m/s

B When $V = 24.5$ then $24.5 = 9.8 \times t$ $\therefore t = \frac{24.5}{9.8} = 2.5$ seconds.

Example 4

Connecting with Geometry: If the height of a right constant cylinder (constant volume) is (h) varies inversely as the square of its radius length r. If the (h) is = 27 cm, when the radius = 10.5 cm, Find (h) when $r = 15.75$ cm.

Solution



$\because v \propto \frac{1}{r^2}$ $\therefore v = m \times \frac{1}{r^2}$ (Where m constant $\neq 0$)

$v = 27$ when $r = 10.5$

$\therefore 27 = m \times \frac{1}{(10.5)^2}$ $\therefore m = 27 \times (10.5)^2$ (1)

Substitute $\therefore v = 27 \times (10.5)^2 \times \frac{1}{r^2}$ from (1)

when $r = 15.75$ cm $\therefore v = 27 \times (10.5)^2 \times \frac{1}{(15.75)^2} = 12$ cm

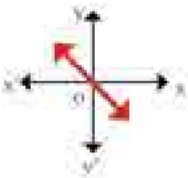
Use the calculator to find the last step as follows:

27 × 10,5 x^2 ÷ 10,75 x^2 =

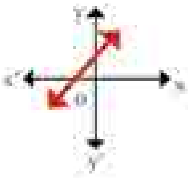
Exercises (2-3)

First: *choose* the correct answers from the given answers:

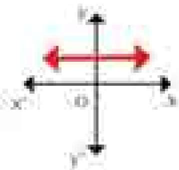
1. **The graphical form represents the direct variation between x and y is :**



A



B



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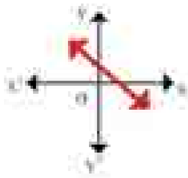


D

2. **The relation represents the direct variation between the two variables x and y which is:**

   A $x y = 5$ B $y = x + 3$ C $\frac{x}{3} = \frac{4}{y}$ D $\frac{x}{5} = \frac{y}{2}$

3. **If y varies inversely with x , and $x = \sqrt{3}$ when $y = \frac{2}{\sqrt{3}}$ then the constant proportional equals:**

   A $\frac{1}{2}$ B $\frac{2}{3}$ C 2   D 6

Second: (Mental Math): From the data of the following table, answer the following questions:

x	2	4	6
y	6	3	2

A Show the kind of variation between y and x   B Find the constant proportion

C Find the value of y when x = 3   D Find the value of x when $y = 2\frac{2}{5}$

General Exercises

1. If the total cost of a trip is (y), some of it is constant (a), and the other is directly proportional with the number of participants (x) then choose the correct answer :

   A) $y = a\,x$ B) $y = \frac{a}{x}$

   C) $y = a + \frac{m}{x}$ (m constant ≠ 0) D) $y = a + m\,x$ (m constant ≠ 0)

2. If $y \propto x$ and $y = 40$ when $x = 14$, *then find* x when $y = 80$.

3. A car moves with a uniform velocity where the distance varies directly with time. If the car covers 150 km in 6 hours, find the distance covered by that car in 10 hours?

4. If the weight of a body on the moon (W) is directly proportional with its weight on the ground (R) if the body weights 84 kg on the ground and its weight on the moon is 14 kg. What will be its weight on the moon if its weight on the ground is 144 kg?

5. If y changes inversely with x and $y = 2$ when $x = 4$. Then find the value of y when $x = 16$

6. If $y \propto x$ *prove that* $y^2 + x^2 \propto y^2 - x^2$

   If a, b, c, and d are in a continued proportional, then prove that:

   A) $\frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{b}{d}$ B) $\frac{2a + 3d}{3a - 4d} = \frac{2a^3 + 3b^3}{3a^3 - 4b^3}$

7. If $\frac{x}{2a + b} = \frac{y}{2b - c} = \frac{z}{2c - a}$ *then prove that* $\frac{2x + y}{4a + 4b - c} = \frac{2x + 2y + z}{3a + 6b}$.

8. ***Connecting with Geometry:*** x, y, z are three proportional sides in a triangle and $x + y = 15$ cm, $y + z = 22.5$ cm. *find* $x : y$.

9. ***Life application:*** Through the interest of the Egyptian authorities with the villages, a budget of 1.85×10^6 pounds was set for one of the villages to build a school, a medical unit and a youth center. If the costs of the school is $\frac{3}{2}$ of the cost of the medical unit and the cost of the medical unit is $\frac{5}{6}$ of the costs of the youth center, what is the cost of each of them?

10. ***Life application:*** If the needed hours to fulfill a work (t) is proportionaly inverse with the number of workers (x) who do the work, If 6 workers fullfiled the work in four hours, what is the time needed for 8 workers to fullfil this work?

Activity

1. *(Mental Math)* **From the data in the following table answer the following questions :**

x	3	8	6	12
y	8	3	4	2

A. ***Show*** and tell the reason why the variation between x and y is an inverse variation.

B. ***Write*** the constant of variation

C. ***Write*** the relation between x and y

D. ***Find*** the value of y when $x = 48$

E. ***Find*** the value of x when $y = 12$

2. **If the rate of success in one of the governrates of the third preparatory is 83% and the rate of success for boys is 79% and the rate of success of girls is 89%. *Find* the rate of success between the number of boys to the number of girls in this governrate .**

Unit Test

1. If $\frac{a+b}{3} = \frac{b+c}{6} = \frac{c+a}{5}$ ***then prove that:*** $\frac{a+b+c}{a} = 7$.

2. **If $y = a - 9$ and $y \propto \frac{1}{x^2}$ and $a = 18$ when $x = \frac{2}{3}$ *then find* the relation between y and x , *then deduce* the value of y when $x = 1$.**

3. **If $\frac{21x - y}{7x - z} = \frac{y}{z}$ *then prove that* $y \propto z$.**

4. **If $x^4 y^2 - 14 x^2 y + 49 = 0$ *then prove that* $y \propto \frac{1}{x^2}$.**

5. ***Connecting with Astronomy:*** **If a weight of a body on Earth (R) directly changes with its weight on the moon (W) , if $R_1 = 182$ kg, $W_1 = 35$ kg, *then find* W_2 and $R_2 = 312$ kg.**

6. ***Connecting with Physics:*** **If the speed of expression v of water to pass through a hose nuzzle inversely changes with the square of the hose nuzzle radius length r and $v = 5$ cm /s when $r = 3$ cm. Find z when $r = 2.5$ cm.**

Unit 3: Statistics

Ice Cream stores produce different kinds of ice cream. The manager conducted a survey on the favorite ice cream the consumers prefer.

Statistics helps you select the sample representing the consumers.

3 - 1 Collecting Data

What you'll learn

- ★ Resources of collecting data
- ★ Methods of collecting data
- ★ How to select a sample
- ★ Types of samples

Key terms

- ☆ Primary resources
- ☆ Secondary resources
- ☆ Method of mass population
- ☆ Method of sample
- ☆ Biased choice
- ☆ Random choice sample
- ☆ Random sample
- ☆ Layer sample

Think and Discuss

The method of collecting data is considered one of the most important phases that statistical research mainly depends on. Collecting data in such scientific methods will lead to get accurate outcomes when doing operations of statistical inference and proper decision making.

1. What are the resources of collecting data?
2. How is the method of collecting data identified?

Resources of collecting data

1 ***Primary resources (Field resources):***

These are the resources which we originally get data through interviewing or questionaires (survey). This type is distinguished by accuracy. However, it needs time and efforts beside it is highly expensive to conduct such a type.

2 ***Secondary resources (historical resources):***

We can get our data from authorities and agancies formally work such as central agency for mobilization and statistics , internet and media This type is a good type of resources such that it saves time and money.

The method of collecting data

The method of collecting data is determined according to the aim and the size of the statistical society under study.

For example: The students of a school represent a statistical society whose value is the student .

First : *Method of mass population :*

It means to collect the data related to the phenomenon of the statistical society. It's used to include all the society such as the population. This type is including all the values and it's unbiased in addition the outcomes are so accurate.

The disadvantages of such a method are ; it needs long time and great efforts. Further more, it costs much money.

Second: ***Mehods of samples:***

It mainly depends upon selecting a sample from the statistical society that it represents.

We conduct reseaches on the sample. The outcomes we get are generalized on the whole society.

Advantages of using methods of samples:

1. It saves time, efforts and money.
2. The only way to collect data about gigantic societies (like fish).
3. The only method to study some limited societies such as:
   - A) Check the patient blood by getting a sample (checking the whole blood leads to death).
   - B) Check the production of a factory producting electric lamps to determine the validity of the lamp. (Know for how long the lamp can be used before getting burned).

Some of the disadvantages of the sample methods are : the outcomes of such type are not accurate if the selected sample doesn't represent all the society well in such a case the sample is **called biased**.

How we select samples and the conditions must be found in getting a sample:

First: the biased selction (samples are not randomly selected)

It means that we select the sample in a way to satisfy the objectives of the research. This is called as the sample delibrate. For example, when we want to know how the students understood a lesson in mathematics we must analyze the outcomes of the test by considering the outcomes of a group of students studied the same topic without the other students this is not a random selection.

Second: Random seclecion (random samples)

It means to select a sample such that the chance of getting any value from the society is equal.

Of the most important types of the random samples :

1. Simple random sample:
   Is the simplest type of samples and it can be get from the homogeneous socities where their selection is related to the size and number of units in the society.

   A) **If the size of the society is small:**
   When we choose 5 students of a 40-student class, then we can prepare a card for each student on which their names or numbers are written, where all the cards are identical, put them back again in the box and draw a card from the box randomly and return the ball back again. Repeat this experiment till you get the sample needed.

B **If the size of the society is big:**

suppose we want to select the sample (5 students) from all the students whose numbers 800. The process of selection will be difficult to be done. So, we number the students from 1 to 800, then use the calculator or excel program to give 10 random digits in the field from 0.000 to 0.999 and take out the decimal point to make the field from zero to 999 you can take out the decimal digits which are more than 800 as follows:

Repeat pressing on  the appearence of numbers will be successive.

2 5 digits unrepreated are enough to give the digits of the sample for the students.

Layer random sample:

When the society needed to be examined is heterogeneous or made up of qualitative sets that are differenct in characteristics, the society is divided into homogeneous sets according to the characteristics forming it. Each set is called a layer and the researcher selects a random sample which each layer is represented according to its size in the society, such as a sample is called the layer sample .

For example: when we want to study an educational level of a society of 400 persons where the ratio of males to females is 3:2 and we want to select a sample of 50 persons, we must select 30 persons from the male layer and 20 persons from the female layer randomly.

Exercises (3-1)

1 ***Compare between*** **the mass population and samples showing the advantages and disadvantages .**

2 **The adminstration of a hotel wanted to conduct a survey to 300 customers on the service level produced. Every customer got a digit from 201 to 500. 10% of them were selected as a random sample to question them about the survice level. Determine using the calculator the digits of the marked customers in this sample.**

3 **At a faculty, there are 4000 university students in the first grade, 3000 in the second grade, 2000 in the third grade and 1000 in the fourth grade if we want to draw a layer sample of 500 students, where each layer is represented in this sample according to the size. Calculate the number of students in each layer in the sample.**

3 - 2

Dispersion

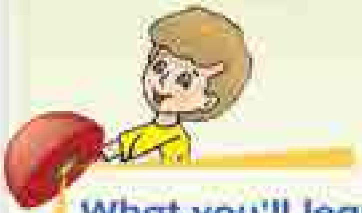
Think and Discuss

You have previously learned the central tendancy (mean - domain - mode) and you used them to calculate a set of data to identify one value describing the trend of these data in centeralization around this value.

If the weekly wages in pounds of two sets of workers A and B in a factory are as follows:

Set A: 170, 180, 180, 230, 240
Set B: 50, 180, 180, 190, 400

1. ***Find*** the mean to the wages of the two sets A and B.
2. ***Compare*** the wages of the two sets A and B. **What do you deduce?**



What you'll learn

- ☆ Dispersions (Range- standard deviation)

Key term

- ☆ Central tendency
- ☆ Mean
- ☆ Dispersion
- ☆ Range
- ☆ Standard deviation

You know that The mean $= \frac{\text{Total of these values}}{\text{Their number}}$

then:

the mean of wages for set A $= \frac{170 + 180 + 180 + 230 + 240}{5}$

$= \frac{1000}{5} =$ LE 200

The mean of wages of set B $= \frac{50 + 180 + 180 + 190 + 400}{5}$

$= \frac{1000}{5} =$ LE 200

Compare the wages of the two sets A and B to find :

1. The mean of wages for set A = the mean of wages of set B = LE 200
2. The median of wages = the mode wage = LE 180 for each set A and B

We notice that :

(1) The wages of the two sets are different but both have the same measures of central tendency.

(2) The wages of set A are close so the values are included between 170 and 240 pounds where the wages of set B are divergent so the values are included between 50 and 400 pounds.

i.e. **The wages of set B is more divergent than the wages of set A.**

So When we compare two sets, we must consider the dispersion of the values of both sets and being divergent from each other :

Dispersion: **to any set of values means divergent or the differences between its values. The dispersion is small if the difference between the values are little whereas th dispersion is great if the difference between the values are very big (if the difference between the values are great). When the dispersion is zero, then all the values are equal.**
i.e. the dispersion is a measure that express how much the sets are homogenous

From the previous, we deduce:

To compare two sets of data or more, we must have a measure to the central tendency and another for dispersion for each set.

Dispersions measurements

1 *Range: (The simplest measure of dispersions)*

It is the diference between the greatest value and the smallest value in the set.
Compare the two sets above :

First set: 51, 53, 55, 57, 58, 60

Second set : 42, 45, 47, 49, 52, 92

We find that the range of the first set $= 60 - 51 = 9$

the range of the second set $= 92 - 42 = 50$

So the second set is more divergent than the first set

Notice that :

(1) The range is the simplest and easiest method of measuring dispersion.

(2) The range is influenced greatly by the outlier, it is clear that the values of the second set disparses in a range of 50 when we remove the last value (92) from and the range $= 52 - 42 = 10$ or $\frac{1}{5}$ of the previous range .

(3) Since the range doesn't influence by any value in the set except the greatest and smallest values, it doesn't give a clear picture to the dispersion of the set.

2 *Standard deviation :*

Is the commonest measure of dispersions and the most accurate (under certain conditions) which is the positive square root to the average of **squares deviations of values from the mean.**

i.e.:

$$\text{Standard deviation } \sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$

where σ denotes to: (sigma) to tell the standard deviation to the society of data.

$\bar{x}$ (x Bar) denotes the mean of the values of society.

n denotes the number of values .

$\sum$ denotes addition.

First : ***calculating the standard deviation to a set of data :***

Example

Calculate the standard deviation for the values : 12, 13, 16, 18, 21

Solution

To calculate the standard deviation , form the table opposite the mean of a set of values

$$\bar{x} = \frac{\text{Total of these values}}{\text{Their numbers}}$$

$$\therefore \bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{12+13+16+18+21}{5} = \frac{80}{5} = 16$$

$$\because \text{The standard deviation } \sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$

	x	$x-\bar{x}$	$(x-\bar{x})^2$
	12	12 - 16 = -4	16
	13	13 - 16 = -3	9
	16	16 - 16 = 0	zero
	18	18 - 16 = 2	4
	21	21 - 16 = 5	25
Sum	80		54

$$\therefore \text{The standard deviation } \sigma = \sqrt{\frac{54}{5}} = \sqrt{10.8} = \simeq 3.286$$

Second: *Calculating the standard deviation to a frequency distribution :*

For any frequency distribution :

$$\text{the standard deviation } \sigma = \sqrt{\frac{\sum (x-\bar{x})^2 k}{\sum k}}$$

where : **x** represents the value or the center of the set ;

k represents the frequency of the value or the set

$\sum k$ is the total of frequency , $\bar{x}$ is the mean $= \frac{\sum x k}{\sum k}$

Example

The following are the frequency distribution for a number of defective units which found in 100 boxes of manufactured units :

Number of defective units	zero	1	2	3	4	5
Number of boxes	3	16	17	25	20	19

Find the standard deviation to the defective units .

Solution

Consider the number of defetive units **(x)** and the number of the corresponding boxes **(k)** to calculate the standard deviation to the defective units form the following table **:**

The mean $\bar{x}$

$$= \frac{\sum x \times k}{\sum k} = \frac{300}{100} = 3$$

The standard variation σ

$$= \sqrt{\frac{\sum (x-\bar{x})^2 k}{\sum k}}$$

$$= \sqrt{\frac{204}{100}} \simeq 1.428 \text{ units}$$

Number of defective units	Number of boxes k	$x \times k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 k$
zero	3	zero	-3	9	27
1	16	16	-2	4	64
2	17	34	-1	1	17
3	25	75	zero	zero	zero
4	20	80	1	1	20
5	19	95	2	4	76
Total	100	300			204

Drill

The following frequency distribution shows the goals scored in a number of football matches:

Number of goals	Zero	1	2	3	4	5	6
Number of matches	1	4	6	9	5	3	2

Find the standard deviation for the numbers of goals.

Example

The following frequency distribution shows the marks of 40 students in an exam:

Sets	0-	4-	8-	12-	16-20	Total
Frequency	2	5	8	15	10	40

Find the standard deviation for this distribution.

Solution

1. Find the centers of sets x

   Then: The center of the first set $= \frac{0+4}{2} = 2$

   The center of the second set $= \frac{4+8}{2} = 6$

   and then record them in the third column.

2. Multiply the centers of sets × its corresponding frequencies. i.e. **x** × **k** and record in the fourth column. Then find the mean $\bar{x} = \frac{\sum x\, k}{\sum k}$

3. Find the deviation of the center of each set (x) from the mean i.e. find $(x - \bar{x})$

4. Find squares of deviations of the center of each set from the mean. i.e $(x - \bar{x})^2$

5. Find the product of the square deviation of the center of each set from the mean × frequency of this set; i.e. $(x - \bar{x})^2 \times k$

6. Calculate the standard deviation $\sigma = \sqrt{\frac{\sum (x-\bar{x})^2\, k}{\sum k}}$

Sets	Frequency (k)	Center of sets (x)	x × k	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 k$
0-	2	2	4	- 10.6	112.36	224.72
4-	5	6	30	- 6.6	43.56	217.80
8-	8	10	80	- 2.6	6.76	54.08
12-	15	14	210	1.4	1.96	29.40
16-20	10	18	180	5.4	29.16	291.60
Sets	40		504			817.6

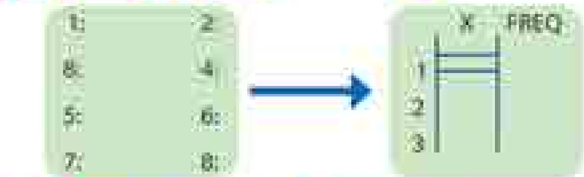
The mean $\bar{x} = \frac{504}{40} = 12.6$

The standard deviation $\sigma = \sqrt{\frac{817.6}{40}} = \sqrt{20.44} \simeq 4.52$ marks

You can use the calculator [Fx-82ES, Fx-83ES, Fx-85ES, Fx-300ES, Fx-350ES] to check the standard deviation.

First: State the calculator on statistical system to enter data

On Mode 2 (Stat) 1 (1-VAR)



Second: Calculate the standard deviation to the frequency distribution **(Example 2)**

1. **Enter the centers of sets 2, 6, 10, 14, 18**

On Mode 2 (Stat) 1 (1-VAR)

2 = 6 = 10 = 14 = 18 =

2. **Go to the initial of the second column (FREQ) and enter the coresponding frequency for each set 2, 5, 8, 15, 10**

3. **Recall sum (standard deviation) then $\sigma \simeq$ 4.521**

Shift 1 5 (VAR) 3 (X σn) =

4. **Go back to the original system and switch off the calculator.**

Notice that :

(1) The standard deviation is affected by the deviations of all the values and its value is affected by the outlier.

(2) The standard deviation has the same measuring units of the original data , so it is used to compare the dispersion of sets which have the same measuring units when the mean is equal in the mean . The set which contains more standard deviation is more dispersion.

Drill

The two frequency tables represent the marks of students of two classes A and B in third prepartory in an exam:

Class A

Sets of marks	0-	10-	20-	30-	40-50	Sum
Number of students	2	5	11	15	7	40

Class B

Sets of marks	0-	10-	20-	30-	40-50	Sum
Number of students	2	3	18	7	10	40

1. ***Represent*** both distribution using the frequency polygon in one figure.
2. ***Find*** the mean and standard deviation for both frequency distributions.
3. Which class is more homogeneous in getting marks?

Exercises (3-2)

1. ***Calculate*** **the standard deviation for the next data:**

   A) 16, 32, 5, 20, 27 B) 72, 53, 61, 70, 59

   C) 15, -12, -9, 27, -6 D) 22, 20, 20, 20, 18

2. **If the standard deviation of a set of data = zero, *what do you* infer?**

3. **The following frequency distribution shows the number of childern of some families in a new city:**

Number of children	Zero	1	2	3	4
Number of families	8	16	50	20	6

Calculate **the mean and standard deviation to the number of children.**

1. The following frequency distribution shows the weights of 200 students in a school:

Weight in kg	35-	45-	55-	65-	75-85	Total
Number of students	20	55	80	30	15	200

Find: A) the mean of students weights.

B) The standard deviation of students weights.

General Exercises

1. *Tell* the proper method for collecting the data in each of the following:
   A) Check the quality of wheat before buying.
   B) Check the salt degree of seawater.
   C) Check the validity of gas pipes before distribution.

2. There is a need to draw a layer sample to represent all the layers according to their sizes of a total 40000 values divided into three layers as follows:

Number of layer	1	2	3
number of values in layer	12000	20000	8000

If the number of values in the first layer is 240, calculate the size of the whole sample.

3. *Calculate* the mean and standard deviation to the following data:
23, 12, 17, 13, 15, 16, 8, 9, 37, 10.

4. The following frequency distribution shows the ages of 10 students:

Ages in year	5	8	9	10	12	Total
Number of children	1	2	3	3	1	10

Calculate the standard deviation to ages in years.

5. The following distribution table shows the amount of gasoline a set of cars consumes:

Number of kilometers per litre	5-	7-	9-	11-	13-	15-17	Total
number of cars	3	6	10	12	5	4	40

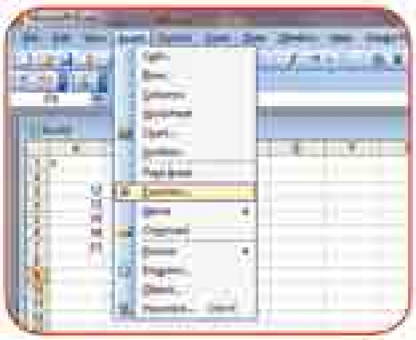
Find the standard deviation to the number of kilometers per litre.

Connecting with technology

Use the computer to calculate the standard deviation.

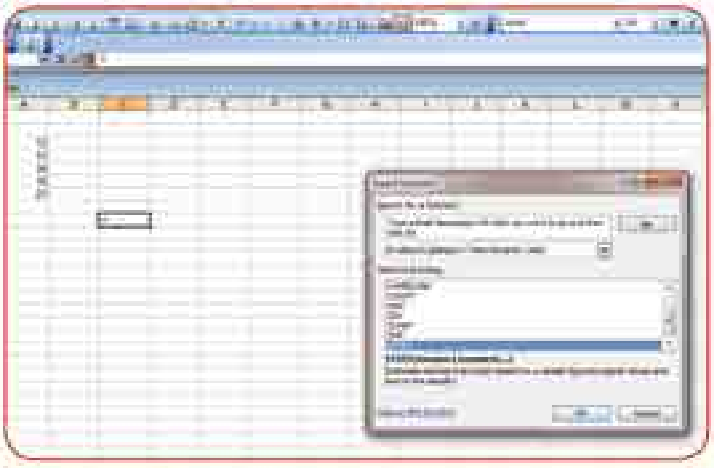
First: (Start) then (programs) (Excel) the following screen appears:

1



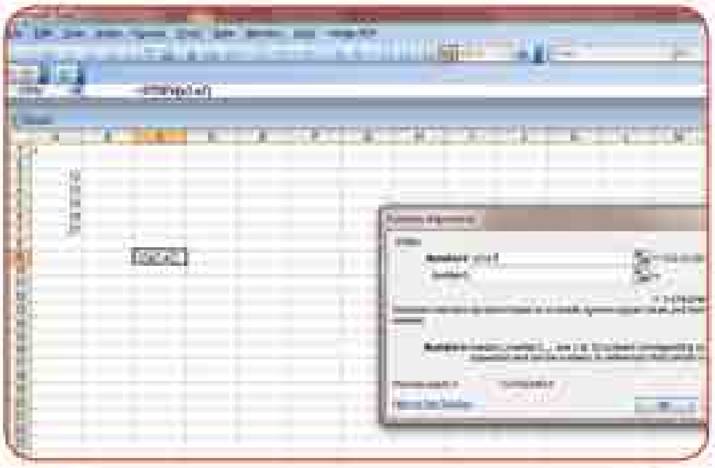
- Enter data of example (1) in the range (A3, A7) as shown
- From (insert) select function (fx) then enter

2



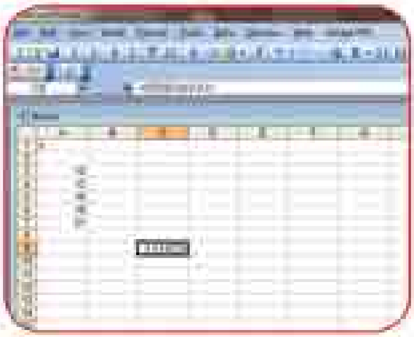
- From the square of searching for data, select the function STDEVP then enter

3



- To calculate the standard deviation to the society of data , determine the range of the veriable (A3, A7) then enter

4



- Notice that the standard deviation to the society of data = 3.286335 is the same as the result in the previous example which the calculator is used

Activity

1. Use the method of samples to select a random sample from your classmates of 10 values. Measure their heights in centimeters and find the average height of your classmates .

Comapre your results and your classmates. Explain your answer.

2. The table opposite shows the temperature in some cities.

   A. Calculate the mean and standard deviation to the maximum temperature.

   B. Calculate the mean and the standard deviation to the minimum temperature .

(You can follow the daily weather reports and calculate the standard deviation and add it to your portfolio)

City	Max	Min
Ismailia	25	11
Suez	26	12
Arish	24	10
Nekhl	24	6
Taba	22	7
Tor	26	16
Hurghada	27	15
Rafah	26	11

Unit test

1. ***Explain*** **briefly the simple random sample explaining how it can be selected.**

2. ***Calculate*** **the mean and the standard deviation for the following data:**

   A. 65, 61, 70, 64, 70, 76, 70

   B. 39, 85, 46, 91, 88, 50, 77

   Which set is more homogeneous?

3. **Calculate the mean and the standard deviation for the following frequency distribution:**

Set	Zero	4-	8-	12-	16-20	Total
Frequency	3	4	7	2	9	25

4. **200 employees were surveyed about their favorite food during break time. Every one was given a digit numbered from 1 to 200 then a sample represent s 10% was selected to be interviewed about their favorite food :**

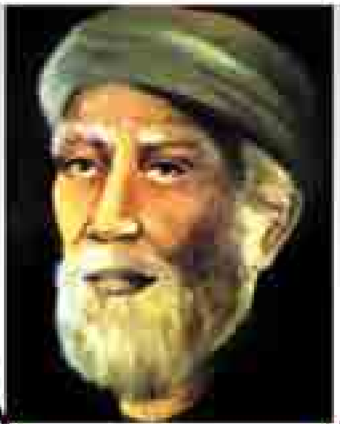
   A. Hot drinks

   B. light meals

   C. soft drinks

   Determine using your calculator the digits of target employees in this sample.

Unit 4 : Trigonometry



Trigonometry is a branch of mathematics that concerned with studying relationships among sides and angles of triangles. Ancient Egyptians were the first to apply the rules of trigonometry in constructing their immortal pyramids and temples as well as applying in astronomy and in calculating geographical distances. Further more Babylonians had also measured the angles in degrees, minutes and seconds. Abou Alryhan Albyrony had settled a table for tangents of angles . Al tousi had deduced that the cosinese of the angles are in proportion with the legs opposite. West civilization learned about what Arab and Muslims wrote through translating the Arab astronomy books by the German Scientist Yohan Muller

Abou Alrayhan Albyrony Was a great scientist born in Algorithm in 973 and died in 1048 AD

4 - 1

The main trigonometrical ratios of the acute angle

What you'll learn

- ☆ Ratios of the acute angle in the right angled triangle.

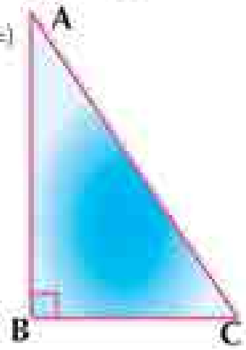
Key Terms

- ☆ Circular measure
- ☆ Sine angle
- ☆ Cosine angle
- ☆ Tangent angle

Think and Discuss

Use the right angled triangle a, b and c shown in the figure opposite ,

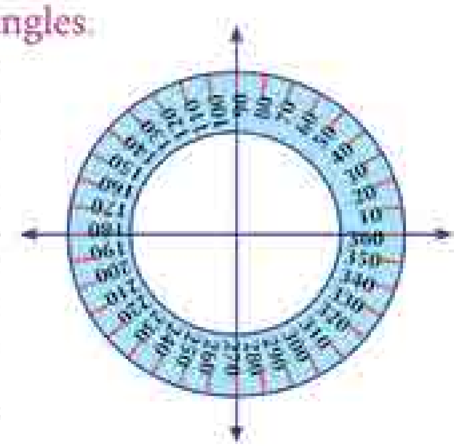
Complete using one of these symbols (> or < or =)



1. If m($\angle$C) > m ($\angle$A) then AB ...... BC
2. $\frac{AB}{AC}$ 1
3. $\frac{AC}{BC}$ 1
4. $\frac{AB}{AC} \div \frac{BC}{AC}$ ...... $\frac{AB}{BC}$
5. $\frac{AB}{AC} + \frac{BC}{AC}$ 1
6. $\frac{(AB)^2}{(AC)^2} + \frac{(BC)^2}{(AC)^2}$ 1

Circular measure of the angles.

We studied that the product of the accummlative angles around a point equals 360°, if you divide the angles into four equal quadrants then a quadrant includes 90° (right angle); and a degree is the circular measuring unit. Similarly, parts of a degree are as follows:



degree = 60 minutes , minute = 60 seconds

35 degrees , 24 minutes ,42 seconds written

as the follows : **35°, 24′ ,42″** you can convert minutes and seconds into parts of the degree in one of the following two ways:

First: Convert $24'$ to minutes $24' = \frac{24}{60} = 0.4$, and convert $42''$ first into minutes then into degrees : $42'' = \frac{42}{60} = 0.7'$

$$0.7' = \frac{0.7}{60} = 0.0116667$$

then the sum is $35^\circ\ 24'\ 42'' = 35 + 0.4 + 0.0116667 = 35.4116667^\circ$

Second: Use the calculator as follows :

The sum is : 35.4116667° **equals** 35 °''' 24 °''' 42 °'''

Similarly, convert the fractions of degree into minutes and seconds.

For example: 54.36° You can convert into degrees , minutes and seconds by using the following keys:

The sum is : $54^\circ\ 21'\ 36''$ °''' SHIFT = 54.36

Drill

1. **Write each of the following angles in degrees:**

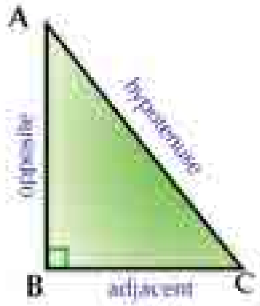
   A. $76^\circ\ 16'$ B. $45^\circ\ 3'\ 56''$ C. $85^\circ\ 38'\ 8''$ D. $65^\circ\ 26'\ 43''$

2. **Write each of the following angles in degrees, minutes and seconds.**

   A. 34.6° B. 78.08° C. 56.18° D. 83.246°

The main trigonometrical ratios of the acute angles:

The figure opposite:



The triangle ABC represents the right angled triangle at B where A and C are two complementary acute angles, the side opposite angle C is called leg opposite, the side adjacent to angle C is called adjacent and the side opposite to the right angle is called hypotenuse.

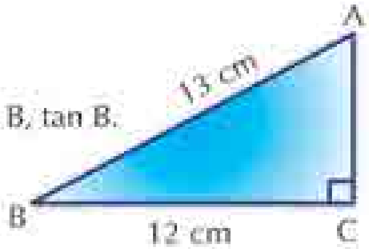
We will know the trigonometrical ratios of the acute angles as the following :

1. **Sine angle**: is denoted by the symbol sin .
2. **Cosine angle**: is denoted by the symbol cos .
3. **Tangential angle**: is denoted by the symbol tan .

sin C	=	$\frac{\text{opposite}}{\text{hypotenuse}}$	=	$\frac{AB}{AC}$
cos C	=	$\frac{\text{adjacent}}{\text{hypotenuse}}$	=	$\frac{BC}{AC}$
Tan C	=	$\frac{\text{opposite}}{\text{adjacent}}$	=	$\frac{AB}{BC}$

Example

1. ABC is a right angled triangle at C, A B = 13 cm, B C = 12 cm



A *Find* the length $\overline{AC}$

B ***Find each of the following:*** sin A, cos A, tan A, sin B, cos B, tan B.

C ***Prove that :*** sin A cos B + cos A sin B = 1

D ***Find :*** $1 + \tan^2 A$

Solution

A $\because$ ABC is a right angled triangle at C $\therefore (AC)^2 = (AB)^2 - (BC)^2$

$\therefore (AC)^2 = (13)^2 - (12)^2 = (13 + 12)(13 - 12) = 25$

$\therefore AC = 5$ cm

B $\sin A = \frac{12}{13}$, $\cos A = \frac{5}{13}$, $\tan A = \frac{12}{5}$, $\sin B = \frac{5}{13}$, $\cos B = \frac{12}{13}$, $\tan B = \frac{5}{12}$

C The right side = sin A cos B + cos A sin B

$\frac{12}{13} \times \frac{12}{13} + \frac{5}{13} \times \frac{5}{13} = \frac{144}{169} + \frac{25}{169} = \frac{144+25}{169} = 1$

D $1 + \tan^2 A = 1 + (\frac{12}{5})^2 = 1 + \frac{144}{25} = \frac{169}{25}$

Drill

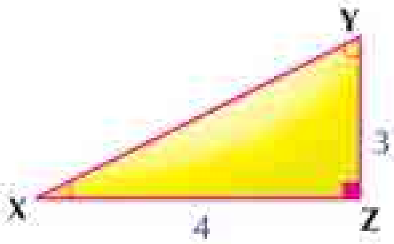
ABC is a triangle in which AB = AC =10 cm , BC = 12 cm, drawn $\overrightarrow{AD} \perp \overline{BC}$, $\overrightarrow{AD} \cap \overline{BC} = \{D\}$

First: find the value of sin (CAD) , cos (CAD) , tan (CAD)

Second: Prove that : A $\sin^2 C + \sin^2 C = 1$ B $\sin B + \cos C > 1$

Exercises (4-1)

1 **In the figure opposite : Complete**



A) $\sin X = ...$ B) $\cos X = ...$

C) $\tan X = ...$ D) $\cos Y = ...$

E) $\tan Y = ...$ F) $\sin Y = ...$

2 If the ratio between two measures of complementary angles as a ratio of 3 : 5, *find* the value of each one by circular measure .

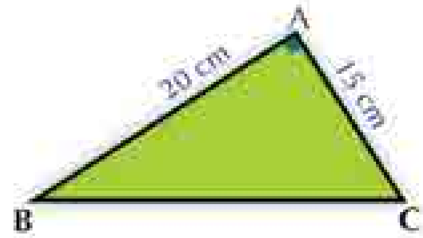
3 If the ratio between two measures of supplementary angles as a ratio 3 : 5, *find* the value of each one by circular measure.

4 If the ratio between the measures of the triangle as a ratio 3 : 4 : 7 *find* the circular measure for each angle.

5 A B C is a right angle triangle in B, A B = 8 cm, B C= 15 cm. Write what each trignometric ratios equal to the following: sin C , cos A , cos C , tan C .

6 ABC is a right angled triangle in B , if $2\,AB = \sqrt{3}\,AC$,
find the main trigonometrical of the angle C .

7 **In figure opposite :**



A B C is a triangle, $m(\angle A) = 90°$, AC = 15 cm ; AB = 20 cm

Prove that : $\cos C \cos B - \sin C \sin B = \text{zero}$

8 X Y Z is right angled triangle at Y, where X Y = 5 cm , XZ = 13 cm

Find **the value of :** A) $\tan X + \tan Z$ B) $\cos X \ \cos Z - \sin X \ \cos Z$

C) $\sin X \cos Z + \cos X \sin Z$

9 X Y Z is a right angled triangle at Z where X Z = 7 cm, X Y = 25 cm.

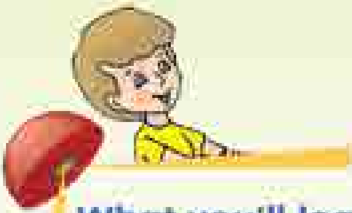
Find the value of each of the following :

A) $\tan X \times \tan Y$ B) $\text{Sin}^2 X + \sin^2 Y$

10 A B C D is an isosceles trapezoid $\overline{AD} \parallel \overline{BC}$, A D = 4 cm, A B = 5 cm where

B C = 12 cm   **Prove that :** $\dfrac{5 \tan B \cos C}{\sin^2 C + \cos^2 B} = 3$

4 - 2 The main trigonometrical ratios of some angles



What you'll learn

- ☆ Finding the trigonometric ratios of angles
- ☆ (30°, 45°, 60°)

Key Terms

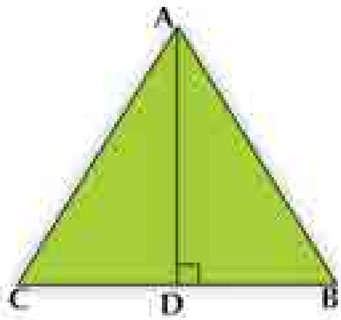
- ☆ Trigonometric ratios
- ☆ Special angles

Think and Discuss

1 In the figure opposite :

A B C is an a equilateral triangle of side length 2L, is and $\overline{AD} \perp \overline{BC}$

Complete:



1. m (∠B) = ...°
2. m (∠B A D) = ...°
3. B D = ... and A D = ... (by L)
4. B D : A B : A D = ... : ...... : ......

From the previous, we notice that :

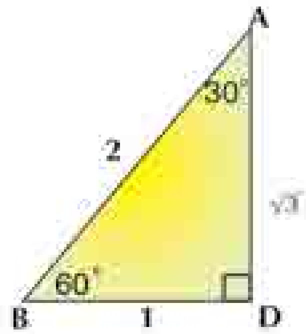
the triangle A B C is 30°, 60° and the ratio between the lengths of the triangle sides are BD : AB : AD = 1 : 2 : $\sqrt{3}$ So you can find the basic trigonometric ratios of the angles 30°, 60° **as follows:**

$\sin 30° = \frac{BD}{AB} = \frac{1}{2}$ and $\cos 30° = \frac{AD}{AB} = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{BD}{AD} = \frac{1}{\sqrt{3}}$

$\sin 60° = \frac{AD}{AB} = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{BD}{AB} = \frac{1}{2}$ and $\tan 60° = \frac{AD}{BD} = \sqrt{3}$



Drill

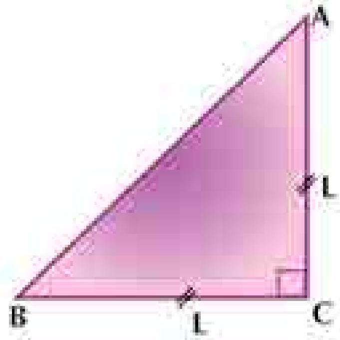
Complete: $\sin 30° = \cos$ ......°, $\tan 30° = \frac{1}{...}$, $\cos 30° = \sin$ ......°

Think and Discuss

1 In the figure opposite:

ABC is an isosceles triangle and a right angled triangle at C. The length of each leg is L.

Complete:



1. m(∠A) = ...°, m (∠B) = ...°
2. ∵ $(AB)^2 = (AC)^2 +$ .... ∴ $(AB)^2 = L^2 +$ ....
3. AC : BC : AB = ... : .... : ....
   ∴ $(AB)^2 = 2L^2$ ∴ $AB = \sqrt{2}\ L$

From the previous, we notice that :

ABC is a triangle in which m (∠A) = m (∠B) = 45° and the ratio between the lengths of its sides are AC : BC : AB = 1 : 1 : $\sqrt{2}$ So you can find the trigonometrical ratios of the angle 45° **as follows:**

$\sin 45^\circ = \frac{AC}{AB} = \frac{1}{\sqrt{2}}$ and $\cos 45 = \frac{BC}{AB} = \frac{1}{\sqrt{2}}$, $\tan 45^\circ = \frac{AC}{BC} = 1$

You can put the previous trigonometrical ratios in the following table:

m angle / ratio	30°	60°	45°
Sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
Cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
Tan	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

Remarks:

1. **From the previous, we find that : (sine)** any angle equals **(cosine)** the supplementary angle of this angle and vice versa .

   for example: sin 30° = cos 60°, cos 30° = sin 60° and sin 45° = cos 45°.

2. For any angle A : $\tan A = \frac{\sin A}{\cos A}$.

Example

1 Find the value of the following :

A $\cos 60° \sin 30° - \sin 60° \tan 60° + \cos^2 30°$

B $\dfrac{\cos^2 60° + \cos^2 30° + \tan^2 45°}{\sin 60° \tan 60° - \sin 30°}$

Solution

A The expression $= \cos 60° \sin 30° - \sin 60° \tan 60° + \cos^2 30°$

$$= \frac{1}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \sqrt{3} + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{2} + \frac{3}{4} = -\frac{1}{2}$$

B The expression $= \dfrac{\cos^2 60° + \cos^2 30° + \tan^2 45°}{\sin 60° \tan 60° - \sin 30°} = \dfrac{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2 + (1)^2}{\frac{\sqrt{3}}{2} \times \sqrt{3} - (\frac{1}{2})} = \dfrac{\frac{1}{4} + \frac{3}{4} + 1}{\frac{3}{2} - \frac{1}{2}} = \dfrac{1+1}{1} = 2$

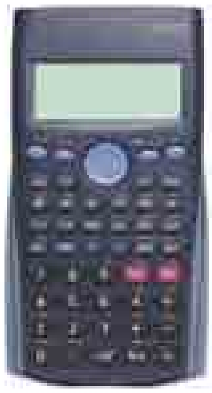
Drill

Prove that:

A $\sin^2 30° = 5 \cos^2 60° - \tan^2 45°$

B $\tan^2 60° - \tan^2 30° = (1 + \tan 60° \tan 30°) \div \cos^2 30°$

Example



2 Find the following trigonometrical ratios :

$\sin 43°$, $\cos 53° 28'$, $\tan 64° 37' 49''$

Roundig the sum to the nearest four decimal numbers .

Solution

Start → sin 43 = $\quad \sin 43° \simeq 0.6820$

Start → cos 53 °''' 28 °''' = $\quad \cos 53° 28' \simeq 0.5953$

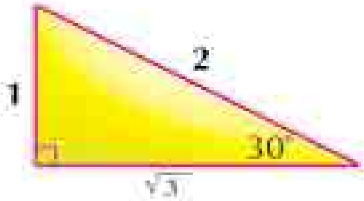
Start → tan 64 °''' 37 °''' 49 °''' = $\quad \text{Tan } 64° 37' 49'' \simeq 2.1089$

Finding the angle given its trigonometrical ratio :

You learned that if you have a given angle, you can find its trigonometrical ratios.

For example: If the measure of an angle is 30° then sin 30° $= \frac{1}{2}$ and similarly, if the angle measure is 33°, then sin 33° = 0.544639035

sin 33° = 0.544639035



Now, we want to identify the angle given its trigonometerical ratio.

for example: If cos C = 0.544639035 find the value of C .

Use the calculator as follows :

Start → SHIFT sin 0.544639035 = °''' 33°

Example

3 **Find m (∠E) in each of the following :**

sin E = 0.6 , cos E = 0.6217 , tan E= 1.0823

Solution

∵ sin E = 0.6 ∴ m (∠E) = 36° 52' 12'' °''' = 0.6 sin SHIFT

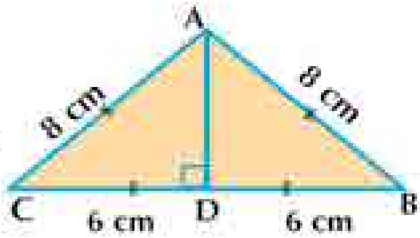
∵ cos E = 0.6217 ∴ m (∠E) = 51° 33' 35'' °''' = 0.6217 cos SHIFT

∵ tan E = 1.0823 ∴ m (∠E) = 47° 15' 48'' °''' = 1.0823 tan SHIFT

4 **Connecting with Geometry:** ABC is an isosceles triangle in which AB = AC = 8 cm and BC = 12 cm .

Find : First: m (∠B)

Second: The area of the surface of the triangle to the nearest two decimal numbers.



Solution

Draw $\overrightarrow{AD} \perp \overline{BC}$

∵ The triangle ABC is an isosceles triangle.

$\therefore$ D the midpoint of $\overline{BC}$ and BD = CD = 6 cm

$\because \cos B = \frac{6}{8} = \frac{3}{4} = 0.75$

Using the calculator :

SHIFT cos 0.75 = °'''

$\therefore m(\angle B) = 41°\ 24'\ 35''$ *(Q.E.D 1)*

To find the surface area of the triangle ; find AD **(From Phythegoran's theorem)**

$\because (AD)^2 = (AB)^2 - (BD)^2$

$\therefore (AD)^2 = 64 - 36 = 28$ $\therefore AD = 2\sqrt{7}$

$\because$ The area of the triangle ABC $= \frac{1}{2} \times BC \times AD = \frac{1}{2} \times 12 \times 2\sqrt{7}$

$= 12\sqrt{7}\ cm^2 \simeq 31.75 cm^2$ *(Q.E.D. 2)*

Another solution for the second part:

$\because \sin B = \frac{AD}{AB}$ $\therefore \sin B = \frac{AD}{8}$

$\therefore AD = 8 \sin (41°\ 24'\ 35'')$ (1)

The area of the triangle ABC $= \frac{1}{2} \times BC \times AD$ **substitute from (1) in this relation**

$\therefore$ The area of the triangle ABC $= \frac{1}{2} \times 12 \times 8 \sin (41°\ 24'\ 35'') \simeq 31.75 cm^2$

Use the calculator as follows :

start → 1 ÷ 2 × 12 × 8 × sin 41 °''' 24 °''' 35 °''' =

Drill

Complete the following :

1. If $\sin X = \frac{1}{2}$ where X is an acute angle then $m(\angle X) =$ ....
2. If $\sin \frac{X}{2} = \frac{1}{2}$ where X is an acute angle then $m(\angle X) =$ ....
3. $\sin 60° + \cos 30° - \tan 60° =$ ....
4. If $\tan (X + 10) = \sqrt{3}$ where X . IS an acute angle then $m(\angle X) =$ ....
5. If $\tan 3X = \sqrt{3}$ where X is an acute angle then $m(\angle X) =$ ....

Exercises (4-2)

1. **Find the value of the following :**

$\sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$

2. **Prove that :**

A. $\cos 60^\circ = 2 \cos^2 30^\circ - 1$

B. $\tan^2 60^\circ - \tan^2 45^\circ = \cos^2 60^\circ + \sin^2 60^\circ + 2 \sin 30^\circ$

3. *Find* **X**

$4X = \cos^2 30^\circ \tan^2 30^\circ \tan^2 45^\circ$

4. *Find* **angle E , where E is an acute angle**

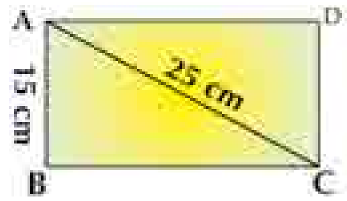
$\sin E = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

5. Connecting with Geometry: in the figure opposite:

ABCD is a rectangle in which AB = 15 cm and AC = 25 cm .

Find : **First:** m($\angle$ACB)

Second : The surface area of the rectangle ABCD .



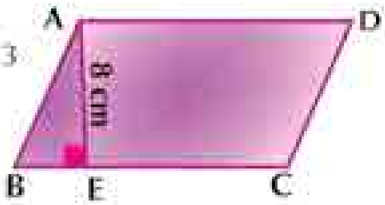
6. Connecting with Geometry : in the figure opposite:

ABCD is a parallelogram of surface area 96 cm^2 , BE : EC = 1 : 3

$\overline{AE} \perp \overline{BC}$ and AE = 8cm

Find: **First:** The length of $\overline{AD}$ **Second:** m($\angle$B)

Third: The length of $\overline{AB}$ to the nearest decimal number ***(Use more than one way)***



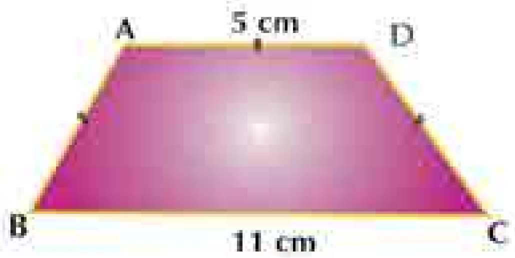
7. Connecting with Geometry: in the figure opposite :

ABCD is an isoceles trapezoid in which

AB =AD= DC = 5 cm and BC = 11cm.

Find : **First:** m ($\angle$B) and m ($\angle$ A)

Second: The area of the isosceles trapezoid ABCD.



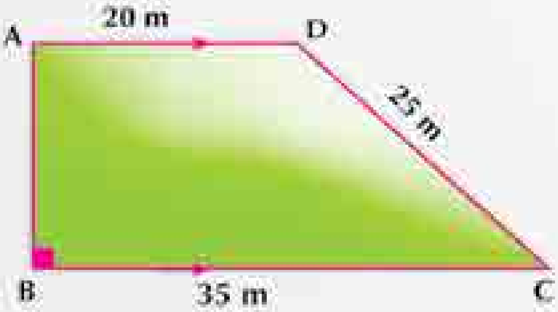
Activity

A trapezoid shaped piece of land ABCD in which $\overline{AD} // \overline{BC}$, m($\angle$B) = 90°, AD = 20 meters

BC = 35 meters and DC = 25meters

R.T.P. : A Find the length of $\overline{AB}$.

B m($\angle$C).



C If the land owner made a circular shaped fountain inside it; What is the largest possible area for the fountain? Find the area of the remaining part of the land. (π = 3,14)

Unit test

1 Prove each of the following equalities :

A $\sin 60° = 2 \sin 30° \cos 30°$

B $\text{Tan } 60° = \dfrac{2 \tan 30°}{1 - \tan^2 30°}$

2 Without using the calculator find the value of X (where X is an acute angle) satisfies each of:

A $\tan X = 4 \cos 60° \sin 30°$

B $2 \sin X = \sin 30° \cos 60° + \cos 30° \sin 60°$

3 ABC is an isosceles triangle in which AB = AC = 12,6 cm and m($\angle$C) = 84° 24'.

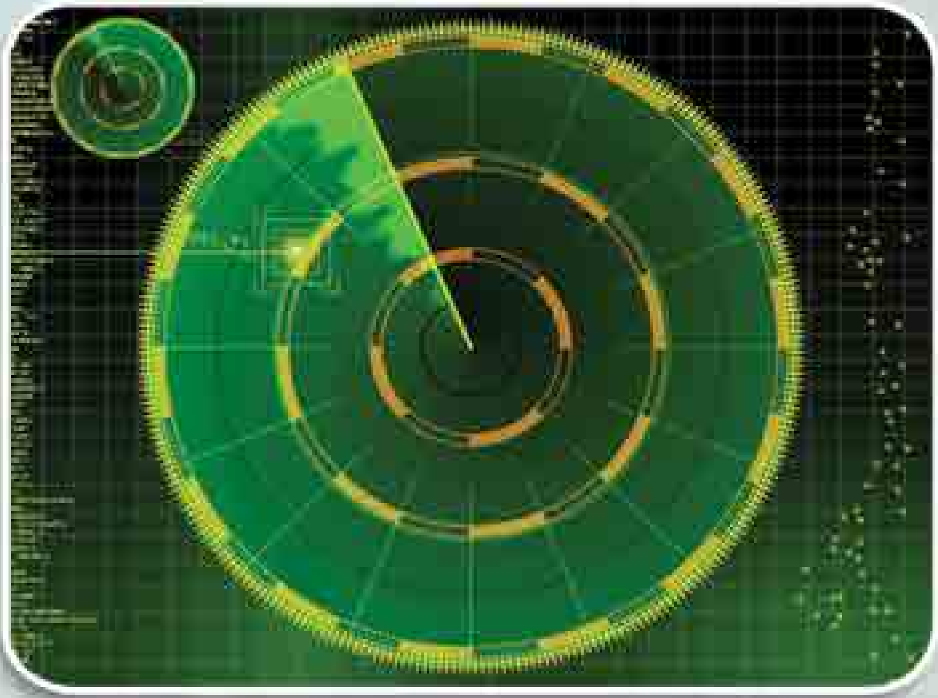
Find the length of $\overline{BC}$ to the nearest decimal number.

4 A B C D is a trapezoid in which $\overrightarrow{AD} // \overline{BC}$, m($\angle$B) = 90°, If AB = 3 cm, AD = 6 cm and BC = 10 cm, **prove that :** $\cos(DCB) - \tan(ACB) = \frac{1}{2}$

5 A ladder $\overline{AB}$ of length 6 meters, its upper edge A lies on a vertical wall and its other edge B on a horizontal floor. If C is the projection of point A on the surface of the floor and its angle of slope on the surface of the floor was 60°, then find the length of $\overline{AC}$.

Coordinate geometry

Unit 5: Coordinate geometry



The Radar is used for identifying the range, height, direction and velocity of moving objects like airplanes and ships.

The radar tower receives the reflected waves. The radar screens can determine the coordinates of the target's location (airplane-ship- ....).

5 - 1 Distance between two points

What you'll learn

- Finding the distance between two points by using the distance rule.

Key terms

- Coordinate plane
- Ordered pair
- Distance between two points.

Think and Discuss

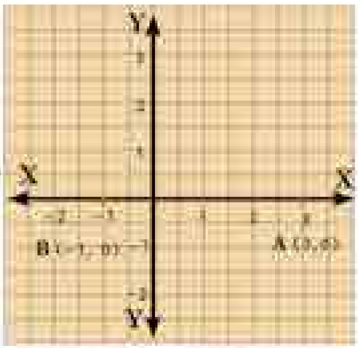
You represented the ordered pair on the coordinate plane.

Now can you find the distance between the pairs of the following points?

1. A (3, 0) , B (-1, 0)
2. C (0, -3), D (0, -1)
3. M (3, 2), N (7, 5)

From the previous, we notice that :

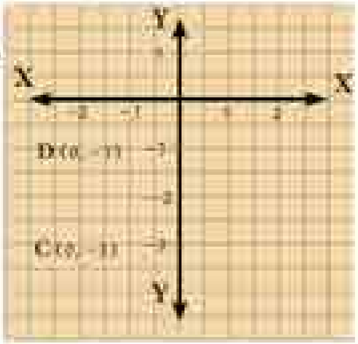
1. The two points A (3, 0), B (-1, 0) are both located on x - axis, so :



$A\,B = |-1 - 3| = |-4|$

So $A\,B = 4$ unit length .

2. The two points C (0, -3), D (0, -1) are both located in the y - axis, so;

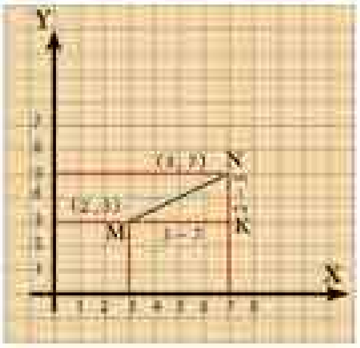


$C\,D = |-3 - (-1)|$

$= |-3 + 1| = |-2|$

$C\,D = 2$ unit length .

3. The two point M (3, 2), N (7, 5) can be represented graphically as in the following figure opposite. To find The length of $\overline{MN}$ we find;



$M\,K = |7 - 3| = 4$ unit length,

$N\,K = |5 - 2| = 3$ unit length .

$\triangle M\,K\,N$ is right angle at K

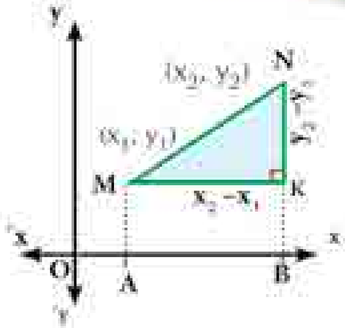
$\therefore (M\,N)^2 = (M\,K)^2 + (K\,N)^2$

(Pythagoren theory)

$(M\,N)^2 = (3)^2 + (4)^2 \quad (L\,M)^2 = 9 + 16$

$(M\,N)^2 = 25 \qquad \therefore (M\,N) = 5$ unit length

In general :



If M (x_1, y_1), N (x_2, y_2) are two points on the coordinate plane

then: $KM = |OB - OA|$

$= |x_2 - x_1|$

$KN = |NB - KB| = |y_2 - y_1|$

$\because \triangle NKM$ is a right angle in K **(pythagoean theory)**

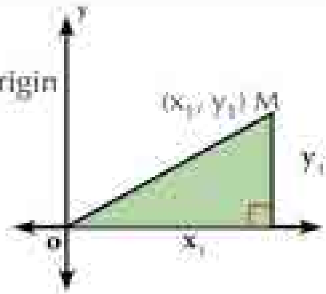
$\therefore (MN)^2 = (KM)^2 + (KN)^2$

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\therefore MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

> The distance between two points (x_1, y_1), $(x_2, y_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
>
> **The distance between two points $= \sqrt{\text{square difference in the x - axis + square differance in y - axis}}$**

Remark:



In the figure opposite the distance of a point M (x_1, y_1) fom the origin point O (0, 0) , $OM = \sqrt{x_1^2 + y_1^2}$

Drill

If A , B , C and D are four given points in the perpendicular coordinate plane, mention the conditions which make those points vertices for each of the following geometrical shapes:

1. Paralleogram
2. Rectangle
3. rhombus
4. Square

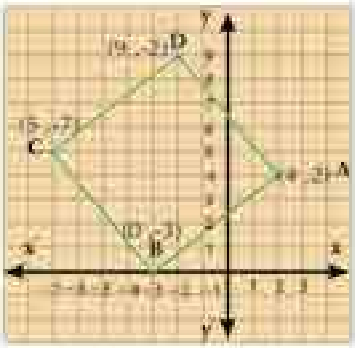
Example

1. ABCD is a quadrilateral where, A (2, 4), B (-3, 0), C(-7, 5) and D(-2, 9). Prove that ABCD is a square.

Solution

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-3-2)^2 + (0-4)^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{41}$

$B\,C = \sqrt{[-7-(-3)]^2 + [5-0]^2} = \sqrt{(-4)^2 + (5)^2} = \sqrt{41}$



$C\,D = \sqrt{[-2-(-7)]^2 + [9-5]^2} = \sqrt{(5)^2 + (4)^2} = \sqrt{41}$

$D\,A = \sqrt{[2-(-2)]^2 + [4-9]^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{41}$

$\because A\,B = BC = DC = D\,A = \sqrt{41}$

$\therefore$ Figure A B C D whether a square ar arhombus to prove that the figure A B C D is a square, find the lengths of the two diagonal $\overline{AC}$, $\overline{BD}$

$AC = \sqrt{[-7-2]^2 + [5-4]^2} = \sqrt{(-9)^2 + 1} = \sqrt{82}$

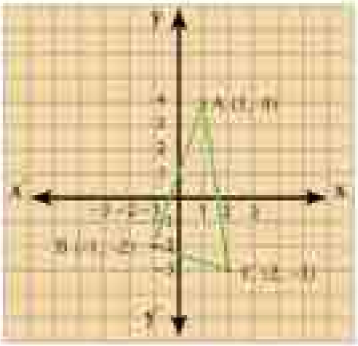
$BD = \sqrt{[-2-(-3)]^2 + [9-0]^2} = \sqrt{(-1)^2 + (9)^2} = \sqrt{82}$

$\because$ $AC = B\,D = \sqrt{82}$ and the sides of the figure A B C D is equal in length

$\therefore$ Figure A B C D is a square.

2 Prove that the triangle of the vertices A (1, 4), B (- 1, -2), C (2, -3) is a right angle. Find its surface area.

Solution



$(A\,B)^2 = (-1-1)^2 + (-2-4)^2 = 4 + 36 = 40$

$(B\,C)^2 = [2-(-1)]^2 + [-3-(-2)]^2 = 9 + 1 = 10$

$(A\,C)^2 = (2-1)^2 + (-3-4)^2 = 1 + 49 = 50$

$(A\,B)^2 + (B\,C)^2 = 40 + 10 = 50$, $(A\,C)^2 = 50$

$\because (A\,C)^2 = (A\,B)^2 + (B\,C)^2$

$\therefore M(\angle B) = 90^\circ$ **(The converse to the pytheogeran theary)**

$\therefore M(\triangle A\,B\,C) = \frac{1}{2} A\,B \times B\,C = \frac{1}{2} \times \sqrt{40} \times \sqrt{10} = \frac{1}{2} \times 2\sqrt{10} \times \sqrt{10} = 10$ square units

3 Prove that the points A (3, -1), B (- 4, 6) and C (2, -2), are located in circle whose center is the point M (-1, 2), , then find the circumference of the circle.

Solution

$A\,M = \sqrt{(-1-3)^2 + [2-(-1)]^2} = \sqrt{(-4)^2 + (3)^2} = \sqrt{25} = 5$

$B\,M = \sqrt{[-1-(-4)]^2 + [2-6]^2} = \sqrt{(3)^2 + (-4)^2} = \sqrt{25} = 5$

$C\,M = \sqrt{(-1-2)^2 + [2-(-2)]^2} = \sqrt{(-3)^2 + (4)^2} = \sqrt{25} = 5$

$\because A\,M = B\,M = C\,M = 5$ $\therefore$ A , B and c are located in a circle whose center is M.

Drill

Prove that the points: A (4, 3), B(1, 1) and C (-5, -3) are collinear.

Complete :

$AB = \sqrt{(1-4)^2 + (1-3)^2} = ...$

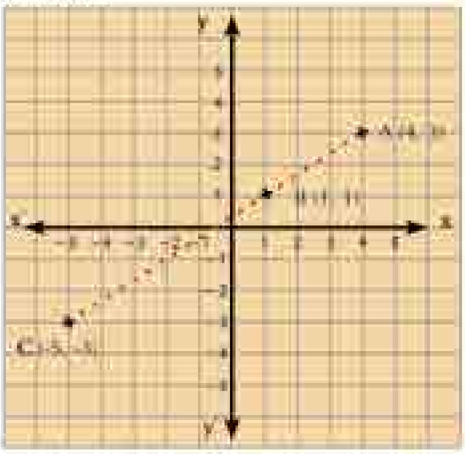
$BC = \sqrt{(-5-1)^2 + (-3-1)^2} = ...$

$AC = \sqrt{(-5-4)^2 + (-3-3)^2} = ...$

$\therefore AB + BC = ... + ... = ...$

$\because AB + ... = AC$

$\therefore$ The points A , B and C are collinear.



Exercises (5-1)

First: Complete the following :

1. The distance between the point (-3, 4) and the point of origin equals ....
2. The distance between the two points (- 5, 0), (0, 12) equals ....
3. The distance between two points (15, 0), (6, 0) equals ....
4. The radius length of the circle of center (7, 4) passing through the point (3, 1) equals ....
5. If the distance between two points (a, 0), (0, 1) is unit length, then a = ...

Second: Choose the correct answer from the given answers:

1. The points (0, 0), (6, 0), (0, 8) :
   - A. form an obtuse triangle
   - B. form an acute triangle
   - C. form a right triangle.
   - D. are collinear.
2. A circle its center is the origin point and radius length 2 units. Which of the following points belongs to the circle?
   - A. (1, 2)
   - B. (- 2, 1)
   - C. $(\sqrt{3}, 1)$
   - D. $(\sqrt{2}, 1)$
3. Show which of the following sets of points are colliner :
   - A. (1, 4), (3, - 2), (- 3, 16)
   - B. (7, 0), (- 3, - 3), (22, 9)
   - C. (- 1, - 4), (1, 0), (0, - 2)
   - D. (- 1, - 4), (1, 0), (0, - 2)

Third: Answer the following questions.

1. Find the value of a in each of following cases :

   A. If the distance between the two points (a, 7), (-2, 3) equals 5

   B. If the distance between the two points (a, 7), (3 a -1, -5) equals 13

2. If A (x, 3), B (3, 2), C (5, 1) and A B = B C then find the value of x.

3. If the distance of the point (x, 5) from the point (6, 1) equals $2\sqrt{5}$, then find the value of x.

4. Tell the kind of each of the following triangles with respect to its angles :

   A. A (3, 10), B (8, 5), C (5, 2)

   B. A (1, -1), B (2, 1), C (-3, -2)

   C. A (3, 3), B (4, -1), C (1, 1)

5. State the kind of triangle whose vertices are the points A (- 2, 4), B (3, - 1), C (4, 5) with respect to its sides .

6. Prove that the triangle whose vertices A (5, -5), B (- 1, 7), C (15, 15) is right angle in B , then find its area .

7. ABCD is a quadrilateral, where points A (5, 3), B (6, -2), C (1, -1), D (0, 4). Prove that ABCD is a rhumbus, then find its area.

8. Prove that the points A (-2, 5), B (3, 3), C (-4, 2) are non collinear, and if D (-9, 4) Prove that the figure A B C D is a parallelogram. .

9. **In the figure opposite :**

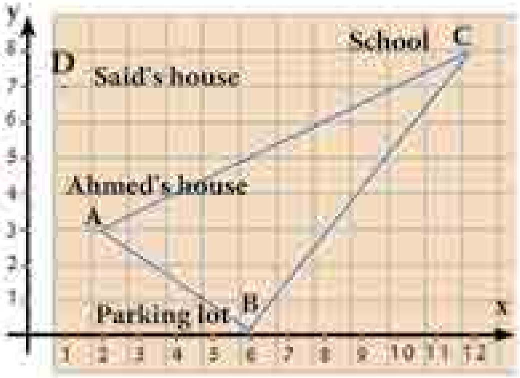
   A. Find the coordinate points which represent the location of Ahmed's house, Saids' house and the parking lot .

   B. The distance of Ahmed's house from the school .

   C. The distance of Said's house from the school.

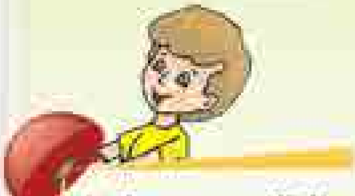
   D. Which is closer to school : Ahmed's house or Said's house?.

   E. Are the two ways of $\overline{AB}$, $\overline{BC}$ perpendcular ? Give the reason.



5 - 2

The Two Coordinates of the midpoint segment



What you'll learn

- ☆ Finding the two coordinates of the midpoint of a straight segment .

Key terms

- ☆ The two ends of the line segment
- ☆ The two coordinates of the midpoint of a straight segment .

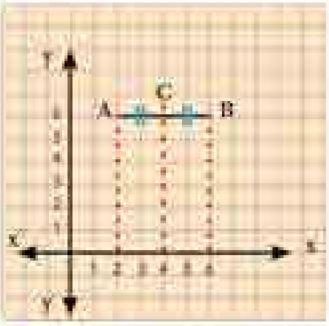
Think and Discuss

On a perpendicular coordinate plane, find the two coordinates of the midpoint on C straight segment $\overline{AB}$:

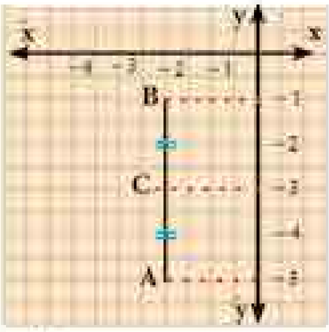
First : A (2, 6) and B (6, 6)

Second : A (-2 , - 5) and B (- 2 , - 1),

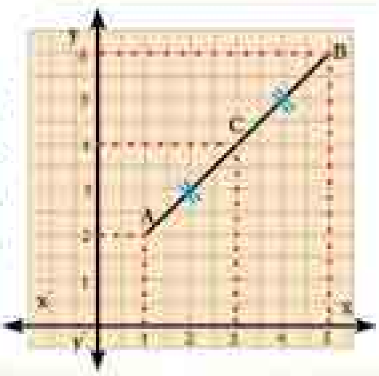
Third : A (1, 2) and B (5, 6)



First: The line segement, which its end are the two points (2, 6), B (6, 6), is parallel to the x-axes and the two coordinate of the point of its midpoint C (4, 6)



Second : The straight segment with the two ends A (-2, -5), B (-2, -1) is parallel to the y-coordinate. The two coordinates of its midpoints C are (-2, -3) .

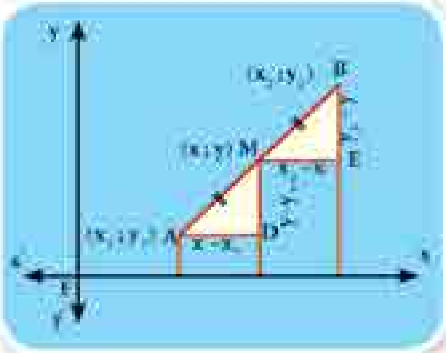


Third : In the figure opposite :

Consider that the C is the midpoint of the straight segment with the two ends A(1, 2), B (5, 6) from the drawing, we find that the two coordinates of C are (3, 4).

i. e C $\left(\frac{1+5}{2}, \frac{2+6}{2}\right)$ i. e. C (3, 4)

In general, you can duduce the law of the coordinate of the midpoint of a straight segment as follows.



If A (x_1, y_1), B (x_2, y_2), M (x, y) where M in the midpoint of $\overline{AB}$ that: $\triangle$ B E M , $\triangle$ M D A are cogruent

we find: that A D = M E

$\therefore x - x_1 = x_2 - x$

$\therefore 2x = x_1 + x_2$ $\qquad \therefore x = \frac{x_1 + x_2}{2}$

Similarly: M D = B E $\quad \therefore y - y_1 = y_2 - y$

$\therefore 2y = y_1 + y_2$ $\qquad \therefore y = \frac{y_1 + y_2}{2}$

$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Example : If C is the midpoint of $\overline{AB}$ and A (3, - 7) , B (- 5, - 3)

Then the coordinates of midpoint of $\overline{AB}$ are $\left(\frac{3 - 5}{2}, \frac{-7 - 3}{2}\right)$ i.e. (- 1, - 5)

Drill

Calculate the coordinates of point C the midpoint of $\overline{AB}$ in the following cases :

1. A(2, 4), B (6, 0)
2. A (7, -5), B (-3, 5)
3. A (-3, 6), B (3, -6)
4. A (7, -6), B (-1, 0)

Examples

1. If C (6, -4) is the midpoint of $\overline{AB}$ where: A (5, -3) then find the coordinates of a point B .

Solution

Consider that B (x_2, y_2), A (5, -3), and the midpoint of $\overline{AB}$ is the point C (6 - 4)

$\because x = \frac{x_1 + x_2}{2}, \; y = \frac{y_1 + y_2}{2}$

$\therefore 6 = \frac{5 + x_2}{2}$ $\qquad \therefore 5 + x_2 = 12$ $\qquad \therefore x_2 = 12 - 5 = 7$

$-4 = \frac{-3 + y_2}{2}$ $\qquad \therefore -3 + y_2 = -8$

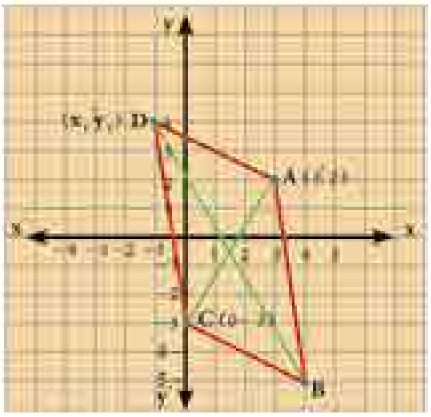
$y_2 = -8 + 3$ $\qquad y_2 = -5$ $\qquad \therefore$ B (7, -5)

2 A B C D is a parallorogram, A (3, 2), B (4, -5), C (0- 3) - Find the two coordinates of the point at which the two diagonals intersect. Then find the coordinates of point D.

Solution

The figure A B C D is a parallelogram , M is the intersection point of its diagonal.

consider D (x_1 , y_1)



$\because$ M is the mid of $\overline{AC}$ $\quad \therefore M(\frac{3+0}{2}, \frac{2-3}{2})$

$\therefore M(\frac{3}{2}, -\frac{1}{2})$

$\therefore M(\frac{4+x_1}{2}, \frac{-5+y_1}{2})$

$\therefore \frac{3}{2} = \frac{4+x_1}{2}$ $\quad \therefore 3 = 4 + x_1$

$\therefore x_1 = -1$

, $\frac{-1}{2} = \frac{-5+y_1}{2}$ $\quad \therefore -1 = -5 + y_1$

$\therefore y_1 = 4$

$\therefore$ The coordinates of the point D are (-1, 4)

Exercises (5-2)

First : Complete

A If the point of the origin is the midpoint of a straight segment $\overline{AB}$, where A (5, -2) then the coordinates of the point B are ....

B If A B = B C = C D, A (1, 3), C (5, 1) where A, B, C, and D are collinear Find :

First : the coordinates of the point B are (........., ...)

Second : The coordinates of the point D are (........., ...)

C $\overline{AD}$ is the median in $\triangle$ A B C, M is the midpoint of $\overline{AD}$

Where A (0, 8), B (3, 2), C (- 3, 6) Find :

First : The coordinates of the point D are (........., ...)

Second : The coordinates of the point M are (........., ...)

Verify by determining the coordinates of the points.

Second:

1. If C in the midpoint of $\overline{AB}$, then find x , y , in each of the following cases:

A	A (1, 5)	B (3, 7)	C (x, y)
B	A (-3, y)	B (9, 11)	C (x, -3)
C	A (x, -6)	B (9, -11)	C (-3, y)
D	A (x, 3)	B (6, y)	C (4, 6)

2. If A (1, - 6), B (9, 2) , then find. the coordinates of the points which divide $\overline{AB}$ into four equal parts in length.

3. Prove that the points A (6, 0), B (2, -4), C (-4, 2) are the vertices of the right angled triangled at B, then find the coordinates of the point D that make the figure A B C D a rectangle .

4. If the points A (3, 2), B (4, -3), C (-1, -2), D (-2, 3) are vertices of the rhombus. Find:
   - A) The coordinates of the the point where the two diagonals intersect the two diagonals .
   - B) The area of the rhombus A B C D.

5. Prove that the points A, (-3, 0), B (3, 4) and C (1, -6) are the vertices of an isosceles triangle of vertex A, then find the length of the drawn straight segment from A perpendicular on $\overline{BC}$.

6. If A (-1, -1), B (2, 3), C (6, 0) and D (3, -4) are four points in perpendicular coordinates plane. **Prove that** $\overline{AC}$ and $\overline{BD}$ bisect each other, then identify the type of the figure.

7. **Prove that** the points A (5, 3), B (3, -2), C (-2, -4) are the vertices of the obtuse triangle at B, then find the coordinates of the point D that makes the figure A B C D a rhombus, and find its surface area.

8. A B C D is a parallelogram where , A (3, 4), B (2, -1), C (-4, -3) . Find the coordinates of D . Take E $\in \overrightarrow{AD}$ where A E = 2 A D . What are the coordinates of the point E?

5 - 3

The slope of the straight line

You know that the slope of the straight line passing through two points (x_1 , y_1), (x_2 , y_2) equals $\frac{y_2 - y_1}{x_2 - x_1}$

What you'll learn

☆ The relation between the slope of two parallel straight lines.

☆ The relation between the slope of two perpendicular, straight lines.

Think and Discuss

Find the slope of the straight line passing through each pair of the following ordered pairs :

First: (3, 1), (4, 2)

Second: (4, 0), (2, 2)

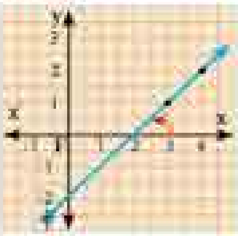
Third: (-1, 3), (2, 3)

Fourth: (2, -1), (2, 3)

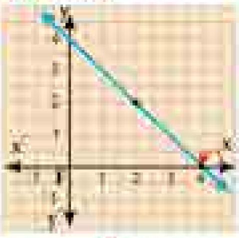
What do you notice?

From the previous, you can draw the straight lines passing through the previous pairs of points in the perpendicular coordinate plane as in the following figure:

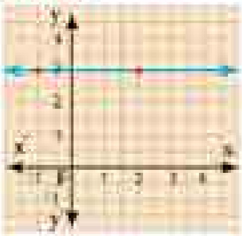
First:



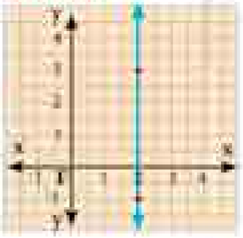
Second:



Third:

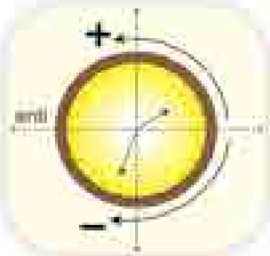


Fourth:



The positive and the negative measure of the angle :

An angle is positive when it is formed by a counter anticlockwise rotation and it is negative when it is formed by a clockwise rotation.



From the previous figures, we deduce that:

The figure number	The slope $\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$	The type of the positive angle that the straight line makes in the positive direction to the x-coordinates	The slope of the straight line
1	$m = \frac{2 - 1}{4 - 3} = 1$	acute	Larger than zero
2	$m = \frac{2 - 0}{2 - 4} = -1$	obtuse	Smaller than zero
3	$m = \frac{3 - 3}{2 + 1} = 0$	zero	equal to zero
4	$m = \frac{3 + 1}{2 - 2}$ (unidentified)	right	unidentified

Key terms

☆ A Positive measure of the angle

☆ A negative measure of the angle

☆ The slope of the straight line

☆ Two parallel straight lines

☆ Two perpendicular straight lines.

We can deduce the slope of the straight line as follows:

Slope of the straight line is the tangent of the positive angle which the straight line makes with the positive direction to x axis.

I.e slope of a straight line = tan E, where E is the positive angle that the straight line makes with the positive direction of the x axis.

Examples

1. Find the slope of the straight line which makes an angle of a measure $56° \ 12' \ 48''$ in the positive direction to the x-axes.
2. Find the measure of the positive angle that the straight line makes to the x - axis if m =1.4865 (where m is the slope) .

Solution

1. $\because m = \tan E$ $\quad\quad \therefore m = \text{Tan } 56° \ 12' \ 48'' = 1.494534405$

Start → tan 56 °′″ 12 °′″ 48 °′″ =

2. $\because m = \text{Tan } E$ $\quad\quad \therefore \text{Tan } E = 1.4865$ $\quad\quad \therefore m(\angle E) = 56° \ 4' \ 13''$

Start → shift tan 1.4865 = °′″

Drill

1. Find the slope of the straight line that makes a positive angle in the positive direction of to the x - axis, its measure:

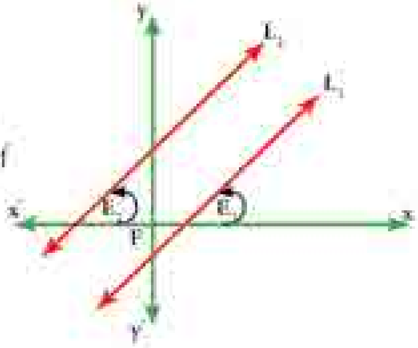
   A) $30°$ B) $45°$ C) $60°$

2. Using the calculator, find the measure of the positive angle made by the straight line of slope (m) in the positive direction of x-axis in the following cases :

   A) m = 0.3673   B) m = 1.0246   C) m = 3.1648

The relation between the slope of the two parallel straight lines.

Think and Discuss



The figure opposite: Represents two parallel straight lines L_1, L_2 with two slpoes m_1, m_2, making two positive angles of measures E_1, E_2 in the positive direction of the x-axes.

Complete the following :

1. $m(\angle E_1) = m(\angle E_2)$ because ....
2. Tan E_1 .... Tan E_2
3. m_1 .... m_2

from the previous, we deduce that :

IF $L_1 // L_2$ **then** $m_1 = m_2$

i.e.: If two lines are parallel, then their slopes are equal and vice versa .

Thus If $m_1 = m_2$ then $L_1 // L_2$

i.e.: If two lines have equal slopes, then the two lines are parallel.

Examples

1. Prove that the straight line passing through two points (-3, -2), (4, 5) is parallel to the straight line that makes with the positive direction to the x-axes an angle of 45 ° measure

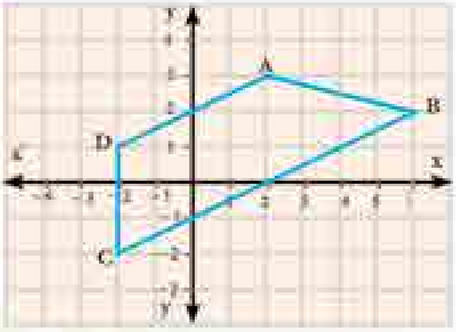
Solution

The slope of the first straight line $(m_1) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-2)}{4 - (-3)} = \frac{7}{7} = 1$

The slope of the second straight line $(m_2) = \text{Tan } 45° = 1$ $\qquad \because m_1 = m_2$

$\therefore$ The two straight lines are parallel.

2. Represent graphically the points A (2, 3), B (6, 2) C (-2, -2) and D (- 2, 1), in the coordinate plane then **prove that** the figure A B C D is trapezoid .



Solution

From the drawing, we find that : $\overline{AD} // \overline{BC}$

To prove that anaclytically, we find the slope of each of both: $\overleftrightarrow{AD}$, $\overleftrightarrow{BC}$.

The slope of $\overline{AD}$ (Let it be m1)

$\because m = \frac{y_2 - y_1}{x_2 - x_1}$ $\therefore m_1 = \frac{3-1}{2+2} = \frac{2}{4} = \frac{1}{2}$

and the slope of $\overline{BC}$ (Let it be M2)

$m_2 = \frac{2+2}{6+2} = \frac{4}{8} = \frac{1}{2}$ $\because m_1 = m_2$ $\therefore \overline{AD} // \overline{BC}$

$\therefore$ The figure A B C D is a trapezoid unles the points A, B, C, D are collinear ..... (1)

$\because$ The slope of $\overline{AB} = \frac{3-2}{2-6} = \frac{1}{-4}$, , the slope of $\overline{CD} = \frac{2+1}{-2+2}$ .... (unknown)

$\therefore$ The two straight lines are not parallel............ (2)

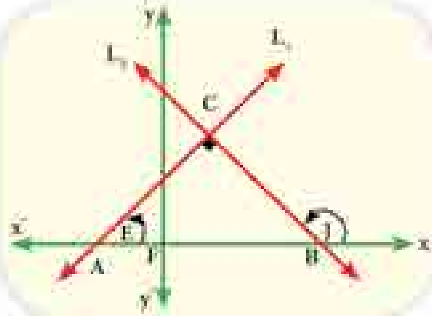
From (1), (2) $\therefore$ The figure A B C D is a trapezoid .

Drill

1. Prove that the straight line passing through the two points (2, 3), (0, 0) is parallel to the straight line passing through the two points (- 1, 4), (1, 7).
2. Prove that the straight line passing through the two points (2, -1), (6, 3) is parallel to the straight line that makes an angle its of 45° measure with the positive direction to the x-axis.
3. If the straight line $\overleftrightarrow{AB}$ // the y-axis where A (x, 7), B (3, 5) , then find the value of x.
4. If the straight line $\overleftrightarrow{CD}$ // the x-axis where C (4, 2), D (- 5,y) then find the value of y.

The relation between the slope of the two perpendicular straight lines.

Think and Discuss



The figure opposite : represents the two straight lines L_1, L_2 which their two slopes are m_1, m_2 where $L_1 \perp L_2$.

Find the relation between $\angle$ E , $\angle$ J)

Then complete the following table :

Values of E	20°	40°	.....	.....	.....
Values of J	.....	.....	140°	150°	.....
Tan E_1 × Tan J_2	.....	.....	.....	.....	.....

From the previous table, we deduce that :

Tan E_1 × Tan J_2 = -1 **i. e** $m_1 \times m_2 = -1$

If L_1, L_2 are two straight lines of slopes m_1, m_2, where $m_1, m_2 \in R^*$
If $L_1 \perp L_2$ then $m_1 \times m_2 = -1$
i. e: The product of multiplying the slopes of the two perpendicular straight, lines = -1 and vice versa, if $m_1 \times m_2 = -1$, then $L_1 \perp L_2$
i. e: If the product of multiplying the slopes of two straights lines = -1, then the two straight lines are perpendiculars.

Examples

1 Prove that the straight line passing through the two points $(4, 3\sqrt{3})$, $(5, 2\sqrt{3})$ is perpendicular on the straight line that makes with the positive direction to the x-axes to an angle of 30° measure.

Solution

Consider that the slope of first straight line is m_1 and the slope of the second straight line is m_2.

$\because m = \frac{y_2 - y_1}{x_2 - x_1}$

$\because m = \tan E$

$\therefore m_1 \times m_2 = -\sqrt{3} \times \frac{1}{\sqrt{3}} = -1$

$\therefore m_1 = \frac{3\sqrt{3} - 2\sqrt{3}}{4 - 5} = -\sqrt{3}$

$\therefore m_2 = \text{Tan } 30° = \frac{1}{\sqrt{3}}$

$\therefore$ The two straight line are perpendicular .

2 If a triangle with vertices y(4, 2), x (3, 5), Z (-5, A) is right angle at y then find the value of A.

Solution

Find the slope of $\overleftrightarrow{xy}$ thus $m_1 = \frac{5-2}{3-4} = \frac{3}{-1} = -3$, find the slope of thus $m_2 = \frac{A-2}{-5-4} = \frac{A-2}{-9}$

$\because \triangle x\,y\,z$ is a right angle at y

$\therefore -3 \times \frac{A-2}{-9} = -1$

$\therefore A - 2 = -3$

$\therefore m_1 \times m_2 = -1$

$\therefore \frac{(A-2)}{3} = -1$

$\therefore A = 2 - 3$

$\therefore A = -1$

Drill

Find the slope of the perpendicular straight line on the straight line through the two points (3, -2), (5, 1) .

Exercises (5-3)

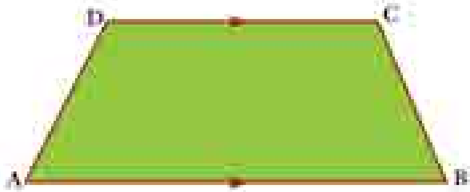
First : Complete the following

1. If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$ then the slope of $\overleftrightarrow{CD}$ equals ....
2. If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{1}{2}$ then the solpe of $\overleftrightarrow{CD}$ equals ....
3. The slope of straight line which is parallel to the straight line passing through the two points (2, 3), (-2, 3) equals ....
4. If the straight line $\overleftrightarrow{AB}$ is parallel to x-axis, where A (8, 3), B (2, K) then K = ...
5. If the straight line $\overleftrightarrow{CD}$ is parallel to the y-axis where C (M, 4), D (-5, 7) then M equals
6. A B C is a right angled triangle in B ,A (1, 4), B (-1, -2) then the slope of $\overleftrightarrow{BC}$ equals .....
7. If the straight, line passing throught the two points (A, 0), (0, 3) and the straight line that makes a triangle its measure is 30° with the positive direction to the x-axes are perpendicular then A = ...

Second :

1. Prove that the straight line passing through the two points A (- 3, 4), C (- 3, - 2)is perpendicular on the straight line passing throught the two points B (1, 2), D (- 3, 2) .
2. If A (- 1, - 1), B (2, 3), C (6, 0) prove that the traingle A B C is right angled triangle in B.
3. If the straight line L_1 passes the two points (3, 1), (2, K) and the straight line L_2 makes with the positive direction to the x-axes a triangle its measure is 45° , then find K, if the two straightline L_1, L_2 :

   A) Parallel B) Perpendicular
4. If the points (0, 1), (A, 3), (2, 5) are located on one straight line, Then find the value of A.
5. Prove that the points A (- 1, 1), B (0, 5), C (4, 2), D (5, 6) are the vertices of the parallelogram.
6. Prove by using the slope that the points A (- 1, 3), B (5, 1), C (6, 4), D (0, 6) are the vertices of the rectangle .
7. In the figure drawn :
   A B C D is trapezoid, $\overline{AB} \parallel \overline{CD}$,
   A (9, -2), B (3, 2), C (x, -x),
   D (4, -3), Find the coordinates of the point C.



8. Prove that the points A (4 , 3), B (7, 0), C (1, -2) are vertices of the triangle, and if the point of D (1, 2) then prove that the figure A B C D is trapezoid and find the ratio between A D , B C.

The Equation of the straight line given its slope and its y - intercept

5 - 4

Think and Discuss

You learned the linear relation between two variables x ; y, it is :

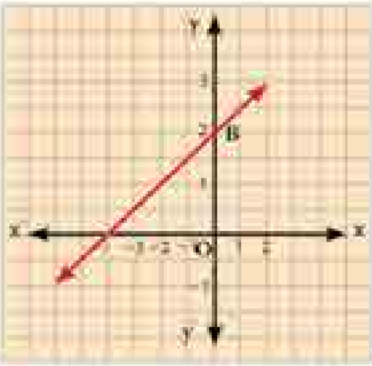
$A x + B y + C = 0$ where A, B (each of both) $\neq 0$

Is represented graphically by a straight line .

What you'll lear

- ☆ Finding the equation of the straight line with given the slope and the intersected part from the y - axis.
- ☆ Finding the equation of the straight line given its slope and its Y-intercept.

Example



Represent the relation :

$x - 2y + 4 = 0$ graphically .

From the graphical figure, calculate:

A The slope of the straight line .

B The lenght of the vertical part included between the origin point and the intesection point of the straight line with y - axis.

Solution to make the drawing easier, select the intersection point of the 2 axes: as follows :

$y = 0$	$\therefore x + 4 = 0$	
$\therefore x = -4$	$(-4 , 0)$	satisfies the relation.
$x = 0$	$\therefore -2y + 4 = 0$	
$\therefore 2y = 4$	$(0, 2)$	satisfies the relation

From the drawing we find that the slope of the straight line

(m) > 0 ..... (why?) thus, $m = \frac{\dots}{\dots} = \frac{\dots}{\dots}$

The distance between the 2 points o and B are called the y - intercept . intercept and is equal to 2 unit lenght and is denoted by the symbol (b).

The previous equation is written as: $y = m x + b$

thus, $2y = x + 4$ and by dividing both sides by 2

$\therefore y = \frac{1}{2} x + 2$

We notice in this form that:

The slope the straight line (m) which is the coefficient of x equals $\frac{1}{2}$, and the lenght of y- intercept $b = 2$ and these are the same results we got the previous drawing .

Key terms

- ☆ Equation of straight line.
- ☆ Slope of a straight line.
- ☆ y - intercept .

Defintion

The equation of the straight line with respect to its slope (m) and the y - intercept (b).

Is y = m x + b where m ∈ R

Notice that : The equation of the straight line is written: ax + by + c = zero, b ≠ 0

In the fromula: y = m x + b as the following :

ax + by + c = zero thus by = ax - c

$\therefore y = -\frac{a}{b} x - \frac{c}{b}$ and it is in the formula: y = m x + c

Where $m = \frac{-a}{b} = \frac{\text{- Coefficient x}}{\text{Coefficient y}}$ Where c is the lenght of the y - intercept .

Examples

1. Find the slope of the straight line 3x + 4 y- 5 = zero in two different methods then find the lenght of the y intercept .

Solution

∵ The equation of the straight line in the formula of ax + by + c = 0, b ≠ 0

$\therefore$ The slope of the straight line $= \frac{-a}{b}$ $\therefore$ The slope of the straight line $= \frac{-3}{4}$

or : it is written in the formula of y = mx + c

$\therefore$ 4 y= -3x +5 $y = \frac{-3}{4} x + \frac{5}{4}$

$\therefore$ The slope of the straight line $= \frac{-3}{4}$ $\therefore$ The length of y - intercept $= \frac{5}{4}$

2. Find the equation of the straight line passing through the point (1, 2) and perpendicular on the straight line passing through the two points A (2, -3), B (5, -4) .

Solution

∵ The slope of the straight line passing through the two points a, b $= \frac{-4-(-3)}{5-2} = \frac{-4+3}{5-2} = \frac{-1}{3}$

thus, the slope of the straight line is perpendicular on = 3

$\therefore$ The equation of the straight line is written in the formula: y= 3x + c

∵ The straight line passes through the point (1, 2) so, it satisfies the equation .

$\therefore 2 = 3 \times 2 + c$

$\therefore c = 2 - 6 = -4$

$\therefore$ The equation of the straight is written in this formula : y= 3x -4

3. If A (- 3, 4), B (5, - 1), C (3, 5) find the equation of the straight line passing through the vertex A and bisecting $\overline{BC}$.

Solution

The midpoint of $\overline{BC} = (\frac{3+5}{2}, \frac{5-1}{2}) = (\frac{8}{2}, \frac{4}{2}) = (4, 2)$

$\therefore$ The slope of the required straight line $= \frac{2-4}{4+3} = \frac{-2}{7}$

$\because y = mx + c$ $\qquad \therefore y = \frac{-2}{7} x + c$

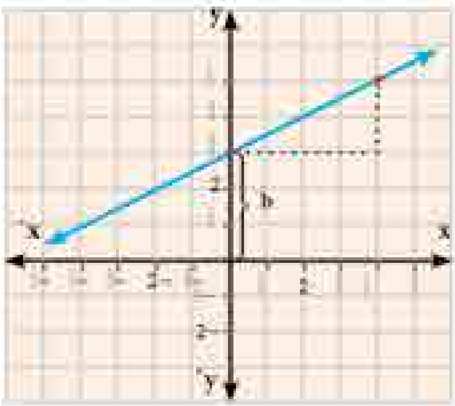
$\because$ The point of A (-3, 4) passes through the straight line, so it satisfies the equation.

$\therefore 4 = \frac{-2}{7} \times -3 + c$ $\qquad \therefore 4 = \frac{6}{7} + c$ $\qquad \therefore c = \frac{22}{7}$

$\therefore$ The equation of the straight line is written as in the formula:$y = \frac{-2}{7}x + \frac{22}{7}$ and by the multyipling two sides in 7

$\therefore 7y = -2x + 22$ $\qquad$ i . e the equation is : $2x + 7y - 22 = 0$

Drill



1. **In the figure opposite, find the following :**
   - A. The slope of the straight line (m) .
   - B. The length of the y - intercept (c).
   - C. The equation of the straight line with given m and c.
   - D. The length of the x intercept.
   - E. The area of the identified triangle by the x - y receprocal.

Exercises (5-4)

1. If $y = mx + b$ represents the equation of straight line with its given slope and the y- intercept. then complete the following :
   - A. The equation of the straight line, when $m = 1$, $c = 3$ is in the form of ....
   - B. The equation of the straight line, when $m = -2$, $c = 1$ is in the form of ....
   - C. The equation of the straight line $m = 3$, $c = 0$, is in the form of ....

2. Find the slope of the straight line and the length of the y - intercept in each of the following :
   - A. $2x - 3y - 6 = 0$
   - B. $5x + 4y - 10 = 0$
   - C. $\frac{x}{2} + \frac{y}{3} = 1$

3. **Find the equation of the straight line in the following cases:**
   - A. When its slope is 2 and intersects a positive part from the y-axis that is equals 7 units.

B) When its slope is equal to slope of the straight line $\frac{y-1}{x} = \frac{1}{3}$ and intersects a part from the negative direction 3

C) Passes by the two points (2, -1) , (1, 1) .

D) The equation of the straight line where m = Zero, c = Zero .

4) Draw the straight line in each of the following:

A) Its slope equals $\frac{-1}{2}$ and intersects a positive part of the y - axis that is equal to one unit.

B) Its slope equals 2 and intersects a negative part of the y - axis equals 3 units.

C) Cuts from the two positive parts of the x - y axes two parts, both lenght are 2 , 3 of the units respectively.

5) The following table represents linear relation:

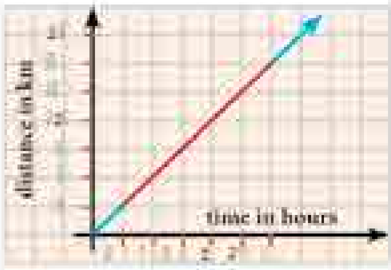
x	1	2	3
y= f (x)	1	3	A

A) Find the equation of the straight line.

B) Find the length of the intersected part from the y - axis.

C) Find the value of A.

6) **In the figure opposite** : The relation between distance the car covers is d in (kilometers), and time the car covers in is t in hour, Find:



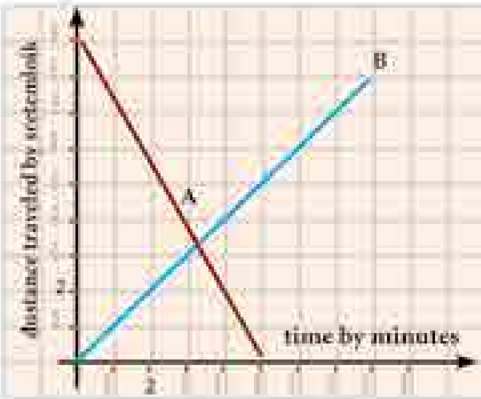
A) The distance traveled in 90 minutes.

B) The time which in the car traveled 150 kilometers.

C) The velocity of the car.

D) The equation of the straight line which converts the relation between d and t.

7) The figure opposite represents the distance traveled (D) in kilometers and the time (T) in minutes of the two objects A and B.



A) If A, B move at the same time?

B) After how many minutes did A, B intersect?

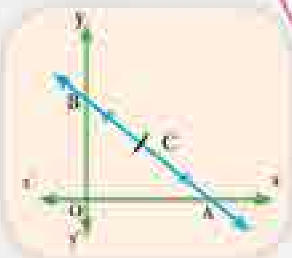
C) What is the velocity of A?

D) Write the equation of the straight line that represents the relation between the distance and the velocity to the movement of the object B?

Activity

1 **In the figure opposite :**

The point C in the midpoint of $\overline{AB}$ where C (4, 3).



First : Complete the following :

A) O A = ... unit length

B) O B = ... unit length

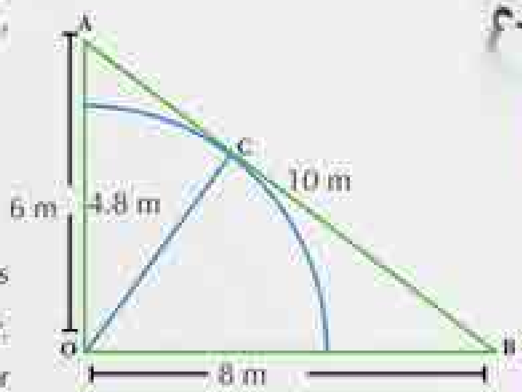
Second : Match between A and B:

Group (A)	Group (B)
a) Slope of $\overleftrightarrow{AB}$	-1
b) Slope of $\overleftrightarrow{OC}$	$\frac{-3}{4}$
c) Slope of $\overleftrightarrow{OA}$	zero
d) Slope of $\overleftrightarrow{OB}$	$\frac{3}{4}$
e) Slope of $\overleftrightarrow{OB}$ × Slope of $\overleftrightarrow{OA}$	1
	unknown

Third : Find the coordinates of the points A , B and O then find the equation of $\overleftrightarrow{AB}$, and the equation of $\overleftrightarrow{CO}$.

Four : Find the length of each of $\overline{CA}$, $\overline{CB}$, $\overline{CO}$

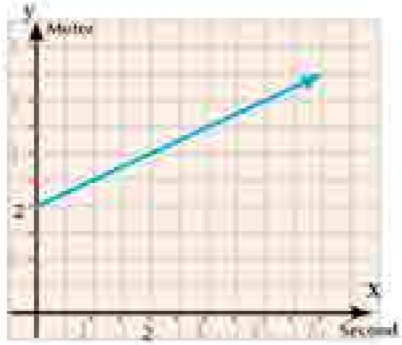
Fifth: Prove in more then or way that C is the center of the circle passing through the points A, O and B.



2 A caw is tied to a point O with rope of 4.8 meters length. If the area is O A B planted with clover. Calculate the area of the cultivated land with clover in which the cow cannot eat, to the nearest meter.

Unit test

1 **The figure opposite :** In the figure opposite a particle moves with a constant speed (v) where the distance (d) is measured by meter and time (t) by second. Find the following:



A The distance at the beginning of moving.

B The velocity of the particle.

C The equation of the straight line which represented the movement of the particle.

D The traveled distance after 4 seconds from the beginning of the movement .

E The time in which the particle covers in distance of 3. 5 meters from the beginning of the movement.

2 Choose the correct answer from the given answers :

1. The slope of the straight line whose equation is $2x - 3y - 6 = 0$:

   A -6 B -2 C $\frac{2}{3}$ D 2

2. If the two straight lines $3x - 4y - 3 = 0$ and $ky + 4x - 8 = 0$ are both perpendicular then k equals:

   A -4 B -3 C 3 D 4

3. If the two straight lines $x + y = 5$ and $kx + 2y = 0$ are both parallel, then K equals :

   A -2 B -1 C 1 D 2

4. The area of the traingle in square unit, identified by straight lines $3x - 4y = 12$, $x = 0$, $y = 0$ equals:

   A 6 B 7 C 12 D 12

5. $\overleftrightarrow{AB}$ is a straight line passes through the two points (2, 5) and (5, 2) which of the following points $\in \overleftrightarrow{AB}$

   A (1, 6) B (2, 3) C (0, 0) D (3, -4)

6. If A (3, 5), B (2, -1) and C (x,y) then the coordinates of the point C. That makes the triangle A B C a right angle triangle at B is:

   A (6, -1) B (-4, 5) C (3, -2) D (8, -2)

3 A (5, -6), B (3, 7) and C (1, -3). Then find the equation of the straight line passes through point A and the midpoint of $\overline{BC}$.

4 Find The equation of the straight line perpendicular to $\overline{AB}$ from its midpoint C where A (1, 3) and B (3, 5) .

5 Find the equation of the straight line passing through the point (3, - 5) and parallel to the straight line $x + 2y - 7 = 0$.

6 Find the equation of the straight line passing through the two points (4, 2) and (-2, -1). Then prove that it passes through the origin point.

7 Find the equation of the straight line which intersects from the x - y axes two positive parts both lengths are 4 and 9 respectively.

8 A B C is a triangle where A (1, 2), B (5, - 2), C (3, 4), D is the midpoint of $\overline{AB}$, drawn $\overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$ and intersects $\overline{AC}$ in E, find the equation of the straight line $\overleftrightarrow{DE}$.

Unit (1)

Answers of page (2)

(A) $a = -5$, $b = 6$

(B) $a - 2 = 2$, $\therefore a = 4$ $b + 1 = -3$, $\therefore b = -4$

(C) $2 - a = 6$, $\therefore a = -4$ $b - 3 = -1$, $\therefore b = 2$

(D) $C - 7 = -2$ $\therefore a = 5$ $b3 - 1 = 26$, $\therefore b^3 = 27$ $B = 3$

Answers of page (4)

(A) $X \times Y = \{(2, 3), (2, 0), (-1, 4), (-1, 0)\}$

(1) $n (X \times z) = 6$ (E) $n (A^2) = 8$

(F) $n (z^2) = 9$

Answers of exercises (7, 8)

First: (1) $a + 5 = 8$ $\therefore a = 3$

$b - 1 = 3$ $\therefore b = 4$

(2) $x^5 = 32$ $y + 1 = 5$

$x^5 = 2^5$ $y + 1 = 3$

$x = 2$ $y = 2$

(4) $n (M) = 3$

Second: (1) a (2) c (3) c (4) b

Answers of exercises page (11)

(4) $x = \{(1, 6), (3, 4), (4, 3), (5, 2)\}$

(5) $x = \{(0, 0), (1, 1), (2, 4), (3, 9)\}$

(6) $\{(1, 1), (2, 8)\}$

Answers of exercises page (14)

(3) $x = \{(2, 10), (2, 16), (2, 24), (2, 30), (5, 10), (5, 30), (8, 16), (8, 24)\}$ not function.

(4) $x = \{(2, 3), (4, 2)\}$ not function.

Answers of page (15)

(1) $F_1 (x)$, $F_4 (x)$

(2) (A) a function of first degree

(B) Constant function

(C) a function of third degree

(D) a function of fourth degree

Answers of page (16)

(B) $D (3) = (3)^2 - 3 (3) - 3 = 9 - 9 - 3 = -3$

$S (3) = 3 - 3 =$ zero $f (3) \times S (3) = -3 \times 0 = 0$

Answers of page (18)

First: (1) -1 (2) -2 (3) 2

Second: (1)

(A) $f (-2) = 3$ (B) $f (-2) = 7$ (C) $f (-2) = 0$

$f (0) = 3$ $f (0) = 3$ $f (0) = -4$

$f (\frac{1}{2}) = 3$ $f (\frac{-3}{2}) = 2$ $f (\frac{1}{2}) = \frac{-15}{4}$

(2) (A) intersection point is (0, 0)

(B) intersection point is (0, 0)

Answers of unit test page (20)

(4) (A) $m = 4$

(C) the area of the triangle whose vertices are A, B and C = 9 square units

Unit (2)

Answers of page (23)

(2) $\frac{3x + 7}{5x + 13} = \frac{3}{5}$

$15 x + 60 = 6 x + 21$

$9 x = 81$ $x = 9$ the two number are : 18, 27

(3) Let number be x

$\frac{13 - x}{17 - x} = \frac{2}{3}$

$147 - 9 x = 138 - 6 x$

$3 x = 9$ $x = 3$

Answers of page (26)

(4) (A) $\frac{2}{x} = \frac{3}{6}$ $x = 3$

(B) $\frac{x}{8} = \frac{x}{12}$ $x = 16$

Answers of page (29)

(3) $5 m + 7 m = 27.6$ $\therefore m = 2.3$

$a = 5 \times 2.3 = 11.5$

$b = 7 \times 2.3 = 16.1$

$c = 3 \times 2.3 = 6.9$

Answers of page (33)

table (1) $x y = 60$ inverse

table (2) $\frac{y}{x} = \frac{2}{3}$ direct

table (3) $\frac{y}{x} = \frac{1}{4}$ direct

Answers of page (34)

first : (1) (D) (2) (D) (3) (c)

second: (A) inverse (B) 12

Answers of exercises page (35)

(1) (D)

(3) $\frac{y_2}{y_1} = \frac{x_1}{x_2}$ $\therefore \frac{y}{1} = \frac{8}{4}$

$\therefore y = \frac{4 \times 2}{16} = \frac{1}{2}$

Answers of unit test page (36)

(3) (2) $\because x - y z = 7 x y - y z$

$\therefore 3 x z = x y$ $\therefore 3 z = y$ $\therefore y \propto z$

(4) $(x^2 y - 7)^2 = 0$ $\therefore x^2 y = 7$

$\therefore y = \frac{7}{x^2}$ $\therefore y \propto \frac{1}{x^2}$

Unit (3)

Answers of exercises page (40)

(1) theoretical.

(2) practical

Answers of exercises page (47)

(4) (A) $\overline{X}_1 = 20$ $\sigma_1 = 9.327$

(B) $\overline{X}_2 = 65$ $\sigma_2 = 7.071$

(C) $\overline{X}_3 = 5$ $\sigma_3 = 15.297$

(D) $\overline{X}_4 = 20$ $\sigma_4 = 1.265$

Unit (4)

Answers of page (53) :

(4) (A) 76.2667 (B) 55.6356

(C) 85.6356 (D) 65.4453

Answers of page (58)

(3) (A) $\frac{3}{4}$ (B) $\frac{3}{5}$ (C) $\frac{3}{4}$ (D) $\frac{3}{5}$ (E) $\frac{4}{5}$

(4) (A) $\tan x + \tan z = \frac{12}{5} + \frac{5}{12} = \frac{169}{60}$

(B) $\sin c \cos b - \sin c \sin b$

$= \frac{5}{13} \times \frac{12}{13} - \frac{12}{13} \times \frac{5}{13} = \frac{0}{169}$

= zero

Answers of page (60)

(1) 30° (2) 120° (3) zero (4) 90° (5) 20°

Answers of unit test page (62)

(4) $\frac{1}{2}$ (3) 3 - 3 meters

Unit (5)

Answers of page (67)

(1) 5 (2) 13

(3) 9 (4) $\sqrt{16 + 9} = 5$

(5) zero

Second: (1) C (2) C

Answers of exercise page (70)

First: (a) (- 3 , 2)

(B) First (3, 2) Second (7, 1)

(C) First (0, 4) Second (0 , 4)

Second:

(1) (A) (2 , 4) (B) $x = 3$, $y = -17$

(C) $x = -15$, $y = 9.5$ (D) $x = 4$, $y = 9$

Answers of page (74)

(2) (a) $m = 26° 1' 5.82''$

(B) $m = 45° 41' 46.11''$

(C) $m = 72° 27' 53.02''$

Answers of page (78)

First:

(1) $\frac{1}{4}$ (2) - 2

(3) zero (4) 3

(5) - 5 (6) - $\frac{1}{3}$

(7) - 9

Answers of page (81)

(3) (A) $y = x + 3$ (B) $y = -2 x + 12$

(C) $y = 3 x$

Answers of unit test page (84)

(2) (A) E (B) C (C) E

(D) C (E) C (F) b

General Exercises on units and model test

Algebra and Statistics

First Complete each of the following

(1) The point (5, - 3) lies in .... quadrant

(2) If (x+ 5 , 8) = (1 , 6 y + x) then x = ... , y = ...

(3) If n (X) = 5, n (X × Y) = 15 then n (Y) = ...

(4) The point (4,0) lies on .... axis

(5) If (5 , x - 7) = (y + 1 , - 5) then x + y = ...

(6) If X × Y = {(1,5) , (1,7) , (2,5) , (2,7) , (3,5) , (3,7)}

then X = ... Y = ...

(7) If f (x) = 5 x - 7 then f (3) = ...

(8) If f (x) = 6 x then f (2) + f (-2) = ...

(9) If f (x) = 3 x + b, f (4) = 13, then b = ...

(10) Function f : R → R such that f (x) = 3 x represented by a straight line passes through the point (-4,............)

(11) The linear function f (x) = x + 7 is represented by a straight line cuts X - axis at the point ....

(12) The linear function f (x) = 2 x - 1 is represented by a straight line cuts y - axis at the point ....

(13) If the point (a, 3) lies on the straight line which represents the function f : R → R where f (x) = x - 5 then a = ...

(14) If f (x) = x - 6 and $\frac{1}{3}$ f (a) = - 2 then a = ...

(15) If X = {1, 3, 5} , f : X → R and f (x) = 2 x + 1 then the range of f = ...

(16) The linear function f (x) = 2 - 3x is represented by a straight line cuts Y - axis at the point ....

(17) If f is function where f : X → Y then X is called .... and Y is called ....

(18) If f is function from set X to set Y then the range of function f ⊂ ....

(19) If f is a function where f (x) = 3 x - 1 is represented graphically by a straight line passes through the point (a, 2) then a = ...

(20) If (2, - 6) ∈ f (x) where f (x) = k x + 8 then k = ...

Second: choose the Correct answer from those given

(1) If n (X^2) = 9 , then n (X) = ...

(a) 3 (b) 6 (c) 18 (d) 81

(2) The point (-3,4) lies in .... quadrant

(a) first (b) second (c) third (d) fourth

(3) If X = {5, 6, 7} then n (X^2) = ...

(a) 3 (b) 6 (c) 9 (d) 12

(4) If X x Y = {(1,3) , (1,4)} then n (X) = ...

(a) 3 (b) 1 (c) 4 (d) 2

(5) If X = {5}, Y = {3} then n (X x Y)

(a) 15 (b) 8 (c) 2 (d) 1

(6) If X = {3, 5, 7} and R is a relation on X then the relation which represents a function is ....

(a) R = {(3,5) , (5,3) , (3, 7)} (b) R = {(3,5) , (5,7)}

(c) R = {3,5) , (5,5) , (7,5)} (d) R = {(3,3) , (3, 5), (3,7)}

(7) If the point (x, 7) lies on Y - axis then 5 x + 1 = ...

(a) zero (b) 1 (c) 5 (d) 6

(8) If R is a function from set X to set Y where

X = {2,5,8}, Y = {3,5} and R = {(2,3), (5,3), (x,3)} then x = ...

(a) 2 (b) 3 (c) 5 (d) 8

(9) If R is a function where R = {(4,3), (5,6), (9,3)} then the range of the function R is ......

(a) {3,4,5,6,9} (b) {4,5,9} (c) {3, 6, 9} (d) {3,6}

(10) If f (x) = 7 x - $\frac{1}{2}$ then f ($\frac{1}{2}$) = ...

(a) 7 (b) $\frac{1}{2}$ (c) $\frac{7}{2}$ (d) 3

(11) If f (x) = 4x + b , f (3) = 15 then b = ...

(a) 156 (b) 3 (c) 4 (d) - 3

(12) If (m,13) satisfies the function f where
$f(x) = 3x + 4$ then m = ...

(a) 6 (b) - 6 (c) 3 (d) - 3

(13) If (2,b) satisfies the function f where
$f(x) = 3x - 6$, then b = ...

(a) Zero (b) 7 (c) 9 (d) 2

(14) If $f(x) = x^2 + 7$ then f (3) = ...

(a) 10 (b) 7 (c) 9 (d) 16

(15) If $f(x) = x^3$ then f (2) + f (-2) = ...

(a) 16 (b) Zero (c) -7 (d) 4

(16) If (2, - 6) satisfies the function f where
$f(x) = kx + 8$ then k = ...

(a) -16 (b) 7 (c) -7 (d) 2

(17) The function f, where $f(x) = 5x$ is represented graphically by a straight line passes through the point ....

(a) (5,5) (b) (0,0) (c) (0,5) (d) (5,0)

(18) If $f(x) = 5x + 4$ is represented graphically by a straight line passes through the point (3,b) then b = ...

(a) 5 (b) 4 (c) 3 (d) 19

(19) If the function f is a function from set X to set Y then the domain of the function is ....

(a) X (b) Y (c) X xY (d) Y x X

Third: Answer the following questions

(1) If X = {0, 1, 2, 3, 4, 5, 6} and R is a relation on X where $_aR_b$ means "a is twice b" for all a, b ∈ X, a ≠ b

(a) Write R and represent it by an arrow diagram.

(b) Is (0,0) ∈ R? (c) Is $_2R_4$?

(d) Find x If $_6R_x$

(2) If X = {2, 4, 8}, Y = {4, 6, 12, 24}, and R is a relation from X to Y such that $_aR_b$ means "b > 2a" for all a ∈ X, b ∈ Y, write R and represent it by an arrow diagram and by a cartesion diagram.

(3) If X = {13, 14, 43, 84}, and R is a relation on X such that $_aR_b$ means "two numbers a and b have the same unit digit" for all a , b ∈ X.
write R and represent it on a lattice

(4) If X = {2, 3, 4, 7}, Y = {1, 2, 3, 4, 7, 8} and R is a relation from X to Y where $_aR_b$ means "a - b is a prime number" for all a ∈ X, b ∈ Y.
write R and represent it by an arrow diagram.

(5) If $X = \{0, 1, 2, 3\}$, $Y = \{-3, -2, -1, 0\}$ and R is a relation from X to Y where $_aR_b$ means "a is additive inverse of b" for all $a \in X$, $b \in Y$, write R and represent it by an arrow diagram and graphically. Is R is a function ? why?

(6) If $X = \{2, 5, 8\}$, $Y = \{10, 16, 24, 30\}$ and R is a relation from X to Y for all $a \in X$, $b \in Y$ where "a is a factor of b" write R and represent it by an arrow diagram. Is R a function? why?

(7) If $X = \{1, 3, 4, 5\}$, $Y = \{1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where $_aR_b$ means "$a + b = 7$" for all $a \in X$, $b \in Y$. write R and represent it by an arrow diagram and by a cartesian diagram. show that R is a function? write its domain and its range.

(8) If $X = \{1, 2, 3\}$, $Y = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{5}\}$and R is a relation from X to Y where $_aR_b$ means "a is the multiplicative inverse of b" for all $a \in X$, $b \in Y$ write R and represent it by an arrow diagram and by a cartesian diagram. Is R a function? why?

(9) If $X = \{1, 2, 4\}$, R is a relation on X such that " a is a multiple of b" for all $a \in X$, $b \in X$, write R and represent it by an arrow diagram and by a cartesian diagram. Is R a function? why?

(10) If $X = \{2, 3, 4\}$, $Y = \{3, 4, 5, 6, 7, 8\}$ and $f : X \rightarrow Y$ where $f(x) = 9 - x$ find the images of the elements of x and represent it by an arrow diagram.

(11) If $X = \{1, 3, 5\}$ and R is a relation on X where $R = \{(a,3), (b, 1), (1,5)\}$ then find the numerical value of the expression $a + b$.

(12) Graph the function f where $f(x) = 4 - x^2$ in the interval $[-3,3]$, from the graph determine:

First: The coordinates of the maximum value of function.

Second : The equation of the axis of symmetry.

(13) Graph the function f, where $f(x) = x(6-x) + 4$ in the interval $[-1,7]$.

(14) Represent the following linear functions graphically

(a) $f(x) = 3x + 1$ (b) $f(x) = 2 - x$

(c) $f(x) = 5x$ (d) $f(x) = -2x$

(15) If the straight line which represents the function $f : R \rightarrow R$, where $f(x) = 6x - a$ cuts Y - axis at the point (b,3) then find the value of a and b

(16) If $X = \{3, 4, 5, 10, 13\}$, $Y = \{4, 5, 7, 8, 9, 19, 25\}$ and R is a relation from X to Y such that $_aR_b$ means "$b = 2a - 1$" for all $a \in X$ and $b \in Y$

(a) Write R (b) Represent R by a cartesian diagram

(c) Find the value of x if $(x, 9) \in R$.

(17) If $X = \{3, 5, 7, 9\}$, $Y = \{a : a \in N, 10 \leq a < 50\}$ and R is a relation from X to Y, where

$R = \{(3, 15), (5, 25), (7, 35), (9, 45)\}$

Write the rule of R

(18) If $X = \{1, 2, 3\}$, $Y = \{1, 3, 6, 9, 13\}$ and R is a relation from X to Y where aRb means "$a = \frac{1}{3} b$"

for all $a \in X$, $b \in Y$

write R and show that it is a function, write its range.

(19) If function $f = \{(1,3), (2,5), (3,7), (4,9), (5, 11)\}$

(a) Write each of domain and range of f.

(b) Write the rule of the function f

(20) If $f : R \to R$ is represented by a straight line cuts Y-axis at (b, 3) where

$f(x) = 6x - a$ find value of $2a + 7b$

Problems on ratio and proportion

1- Complete the fllowing:

1. If $3a = 4b$ then $a : b =$ .... : ....

2. If 3, 4, c and 8 are proportional then $c =$ ....

3. The proprtional mean of $3ab^2$ and $27a^3b^2$ is ....

4. If $\frac{x}{3} = \frac{y}{5}$, then $\frac{3x}{5y} =$ ....

5. If 9, $2x$, $\frac{1}{y^2}$ are proprtional quantities then $xy =$ ....

6. If $4x^2 - 12xy + 9y^2 = 0$ and $x \in R$, $y \in R$, $y \neq o$ then $\frac{x}{y} = \frac{\cdots\cdots}{\cdots\cdots}$

7. If $\frac{a}{b} = \frac{2}{3}$ and $\frac{a}{c} = \frac{3}{5}$ then $a : b : c =$ .... : .... : ....

8. If $\frac{a}{b} = \frac{7}{2}$ then $\frac{a - b}{a + b} = \frac{\cdots\cdots}{\cdots\cdots}$

9. $\frac{x}{6} = \frac{y}{5} = \frac{z}{4} = \frac{\cdots\cdots}{11} = \frac{2y + z}{\cdots\cdots}$

10. If 1, x, 9, y are in continued proportion then $x =$ ...., $y =$ ....

2- Choose the correct answer from those given:

1. The third proportion of the two numbers 3 and 6 is ....

(a) $\frac{1}{2}$ (b) 2 (c) 9 (d) 12

2. If 2, 6, x + 15 are proportional then x = ...

(a) 1 (b) 2 (c) 3 (d) 4

3. If a , b, 2 and 3 are proportional, then $\frac{a}{b}$ = ...

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

4. If $\frac{9}{a^2} = \frac{4}{b^2}$ (where a and b ≠ 0) then $\frac{a}{b}$ = ...

(a) $\frac{2}{3}$ (b) $\pm\frac{3}{2}$ (c) $\pm\frac{2}{3}$ (d) $\pm\frac{4}{9}$

5. The second proportion of the quantities $12ab^2$, ... $21ab$, $14b^2$

(a) $8ab^2$ (b) $8b^3$ (c) $24ab$ (d) $24b^4$

6. If $\frac{x}{y} = \frac{z}{\ell}$ which of the following is true ?

(a) $\frac{x}{\ell} = \frac{y}{z}$ (b) $\frac{x}{z} = \frac{\ell}{y}$ (c) $\frac{x}{y} = \frac{\ell}{z}$ (d) $\frac{x}{z} = \frac{y}{\ell}$

7- The number which added to each of the numbers 1, 3, 7, 15 respectively to be in continued proportion is ....

(a) 1 (b) 2 (c) 3 (d) 4

(8) If $\frac{a}{2} = \frac{b}{3}$ then $\frac{b-a}{b+a}$ equals.........

(a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

(9) If $\frac{x}{2} = \frac{y}{3} = \frac{4x-2y}{z}$, then z = ...

(a) - 2 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 2

(10) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = m$ (where $m \in R^*$), then $\frac{a\,c\,e}{b\,d\,f}$ equals........

(a) m (b) 3m (c) m^3 (d) $3m^3$

3- If the following sets of numbers are proportional, then find the values of x .

(a) 8, x, 4, 5 (b) 11, 3, x, 6 (c) 6, 24, 1, x

(4) Find $x : y : z$ in each of the following

(a) $\frac{x}{y} = \frac{3}{5}$ and $\frac{y}{z} = \frac{4}{7}$ (b) $\frac{x}{y} = \frac{4}{5}$ and $\frac{x}{z} = \frac{3}{7}$

(5) If $\frac{a}{b} = \frac{2}{5}$, then find the value of each of the following ratios

(a) $\frac{a + b}{b}$ (b) $\frac{a}{b - a}$ (c) $\frac{b - a}{b + a}$ (d) $\frac{7a - 2b}{3a + 2b}$

(6) If $\frac{a}{2} = \frac{b}{3} = \frac{2a - b}{m}$, then find the value of m

(7) If $\frac{a}{b - a} = \frac{c}{d - c}$, then prove that a, b, c and d are proportional

(8) If b is the middle proportional between a and c, then prove that.

(a) $\frac{a^2}{b^2} = \frac{b^2}{c^2} = \frac{2a}{c}$ (b) $\frac{a + b + c}{a^{-1}+b^{-1}+c^{-1}} = b^2$

(9) If $\frac{x}{3} = \frac{y}{4} = \frac{7}{5}$, then Prove that:

(a) $\frac{2y - z}{3x - 2y + z} = \frac{1}{2}$ (b) $\sqrt{3x^2 + 3y^2 + z^2} = 2x + y$

(10) If a - 1, a + 1, b - 2, b + 2 are in proportion, then find $\frac{a}{b}$, then prove that $\frac{a + b}{a+b-3} = \frac{3a}{5a-b-3}$

(11) If $\frac{a}{b} = \frac{1}{3}$, $\frac{a}{c} = \frac{1}{9}$ and a + b + c = 26, then find each of a , b and c.

(12) If x, y, z, ℓ are proportional quantities, then prove that :

(a) $\frac{x + y}{z + \ell} = \frac{2x^2 - 3y^2}{2z^2 - 3\ell^2}$ (b) $\sqrt[3]{\frac{5x^3 - 3y^3}{5y^3 - 3\ell^3}} = \frac{x + z}{y + \ell}$

(13) If $\frac{x + y}{\ell + m} = \frac{y + z}{m + n} = \frac{z + x}{n + \ell}$, then prove that : $\frac{x}{\ell} = \frac{y - x}{m - \ell}$

(14) If $\frac{x}{2a+b} = \frac{y}{2b-c} = \frac{z}{2c-a}$, then prove that : $\frac{2x+y}{4a+4b-c} = \frac{2x+2y+z}{3a+6b}$

(15) If $\frac{x+y}{7} = \frac{y+z}{5} = \frac{z+x}{8}$, then prove that : $\frac{x+y+z}{x-z} = 5$

(16) Find the number that should be added to each of the numbers 7, 9 , 12, 15 to be proportional.

(17) Two positive integer numbers, the ratio between them is 3 : 7 and if we subtract 5 from each of them the ratio becomes 1 : 3, find the two numbers.

(18) Find the positive mumber that if we add its square to each term of the ratio 7 : 11 it becomes 4:5

Exercises :

1- Complete the following:

1. If $y = 3x$ then $y \propto$ ....

2. If $xy - 7 = 0$ then $y \propto$,..........

3. If $y \propto x$ and the variable x took the two values x_1 and x_2 and the variable y took the two values y_1 and y_2 respectively then $\frac{x_2}{x_1} = \frac{........}{........}$

4. If $y \propto x$ and $x = 1$ as $y = 4$ then the constant of variation is ....

5. If $y \propto x$ and $y = 2$ when $x = 4$ then $y =$ x

6. If y varies inversely as x and $y = 2$ when $x = \frac{1}{2}$ then $y = \frac{........}{x}$

7. If $x^2y^2 - 4xy + 4 = 0$ then $y \propto$ ....

8. If $y^2 - 6xy + 9x^2 = 0$ then $y \propto$ .....

9. If $y \propto \frac{1}{x}$ and the variable x took the two values x_1 and x_2 and the variable y took the two values y_1 and y_2 respectively then $\frac{x_2}{x_1} = \frac{........}{........}$

10. If $y \propto x$ and $y = 1$ when $x = 4$ then $y =$ .... when $x = 8$

2- Choose the correct answer from those given

1. The relation which represents direct variation between the two variables x and y is

(a) $x\,y = 7$ (b) $y = x + 2$ (c) $\frac{x}{3} = \frac{4}{y}$ (d) $\frac{x}{5} = \frac{y}{2}$

2. If y varies inversely as x and if $x = \sqrt{3}$ as $y = \frac{2}{\sqrt{3}}$, then the constant of variation equals:

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 2 (d) 6

3. If $y - x = \frac{1}{x} - \frac{1}{y}$ where $x \neq y \neq 0$ then

(a) $y \propto x + 1$ (b) $y \propto x$ (c) $y \propto \frac{1}{X}$ (d) $y \propto \frac{1}{x^2}$

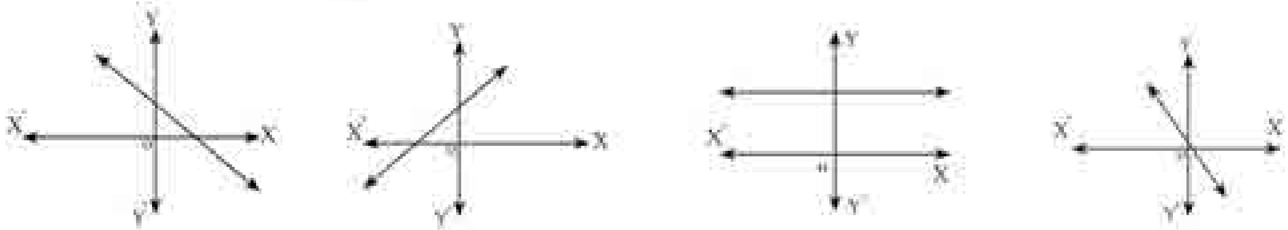
4. If the total cost (y) for a certain trip and if the some of this total cost is constant (a) and the other changes with the number of participated (x), which of the following relations is correct:

(a) $y = a\,x$ (b) $y = \frac{a}{x}$

(c) $y = a + \frac{m}{x}$, (m is a constant $\neq$ o)

(d) $y = a + m\,x$, (m is a constant $\neq$ o)

5. The graph which represents the direct variation between x and y is ....



3- Show which of the following tables represent direct variation, inverse variation or neither-nor, state the reason in each case:

x	y
3	20
5	12
4	15
6	10

x	y
2	9
4	18
6	54
16	72

x	y
5	9
10	18
15	27
25	45

x	y
3	6
-2	-9
-18	1
9	-2

4- If y varies as x and $y = 10$ when $x = 7$, find x when $y = 20$

(5) If y varies inversely as X and y = 10 when x = 3 , find y when x = 5

(6) If $y \propto x$ and y = 20 when x = 7 find the relation between x and y then find the value of y when x = 14

(7) If $y \propto \frac{1}{X}$ and $x = 2\frac{4}{5}$ when $y = \frac{4}{7}$, then find the relation between x and y then find also the value of y when $x = 3\frac{1}{5}$

(8)If y = 3 + a and a $\propto \frac{1}{x}$ if y = 5 when x = 1, then find the relation between x and y and find y when x = 2

(9) Let y = a + 7 and a $\propto \frac{1}{x^2}$ if a = 18 when $x = \frac{2}{3}$ find the relation between y and x , then deduce the value of y when x = 6

(10) If $\frac{21x - y}{7x - z} = \frac{y}{z}$, then prove that $y \propto z$

(11) From the data of the following table answer the following questions

x	2	4	6
y	6	3	2

(a) Identify the kind of variation whether it is direct or inverse

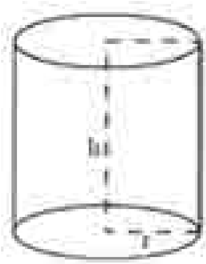
(b) Find the constant of variation

(c) Find the value of y when x = 3

(d) Find the value of x when $y = 2\frac{2}{5}$

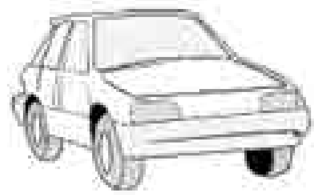
(12) A car moves with constant velocity such that the covered distance varies directly as the time if the car covered 90 km within one and half an hour. write down the relation between the covered distance and the time, then find the covered distance within $2\frac{1}{2}$ hours.

(13) If the light tension (t) of a lamp varies inversely as the square of the distance (d) between the lamp and a pupil studies his lessons at a distance of 12 metres . If the tension of light is weak, then what is the distance which the lamp should be far from the pupil in order that the light tension becomes 4 times what it was before?



(14) If the height of a right circular cylinder (h) of a constant volume varies inversely as the square of the radius length of its base (r) and h = 18 cm when r = 7 cm . find the height h when r = 10.5 cm

15- A car of mass 3 ton moves with uniform velocity under resistence varies as its velocity. If the resistence was 6 kg. weight / ton of the mass of the car when the velocity was 50 km/H find the velocity of the car if the resistence becomes 27 kg. weight / ton



Problems on statistics

(collecting data and dispersion)

First: Complete the following :

1- The statistical sample is a part of ....

2- From the means of collecting data are .... and ....

3- The arithmetic mean is one of measures of .... while the range is one of measures of ....

4- The difference between the greatest value and The smallest value of a set of data is ....

5- The arithmetic mean of a set of values of individuals equals..........

6- The positive square root of the mean of the squares deviations of values from its arithmetic mean is called ....

7- The range of the set of the values 5 , 14 ,4 ,21 , 16 and 12 is ....

8- The standard deviation of the set of the values 3 , 12 , 17 ,28 and 30 equals ....

9- The difference between the greatest individual and the smallest individual of a set of values is called ....

10- If 78 is the greatest individual of a set of individuals and its range is 39 then the smallest individual of this set equals ....

Second: Choose the correct answer from those given:

1- Selecting a sample of layers of a statistical society is called .... sample.

(a) random (b) class (layer) (c) deliberate (d) bunch

2- The difference between the greatest value and the smallest value of a set of individuals is called ....

(a) the range (b) the arithmetic mean

(c) the median (d) the standard deviation

3- The range of the set of the values 7 ,3 ,6 ,9 and 5 equals

(a) 3 (b) 4 (c) 6 (d) 12

TEST

4- The arithmetic mean of the set of the values 7 ,3 ,6 ,9 and 5 equals...........

(a) 3 (b) 4 (c) 6 (d) 12

5- If $\sum (x - \bar{x})^2 = 36$ for a set of values whose number is 9 then σ = ...

(a) 2 (b) 4 (c) 18 (d) 27

Answer the following questions:

1- A school has 360 boys and 480 girls, the school wanted to hold a survey on a smaple of 35 students (boys and girls) such that each layer should be represented due to its size. Calculate the number of individuals of each layer of the sample.

2- A factory has 125 workers ,75 technician and 50 ingineers. It is wanted to take a sample of layers of size 50 individuals such that it represents each layer due to its size. Calculate the number of individuals of each layer in the sample.

3- The following table represents the number of students in one of university colleges.

The grade	1^{st}	2^{nd}	3^{rd}	4^{th}
Number of the students	900	800	700	600

It is wanted to draw a sample of layers of size 120 randomly such that it represents each layer due to its size. Calculate the number of each layer of the sample .

4- It is wanted to draw a random layer sample to represent each layer due to its size from a society of 10 000 individuals and it is divided into two layers as follows

Layer	first	second
number of the layer	3000	7000

If the number of individulals which represents the first layer of the sample equals 90 individuals find the total number of the sample.

5- Calculate the range of each of the following data

(a) 7 ,16,14,9,5 (b) 13 ,9 ,25,19,29,10

6- Calcualte the arithmetic mean and the standard deviation of each of the following data.

(a) 15,30,6,18,16 (b) 68 ,54 ,63,70,45

(c) 70 ,64 ,61 ,65 ,70 ,76 ,70 (d) 23,12,17,13,15,16 ,8,9,37,10

7- If the marks of a student in the mid-year test of 5 subjects are as following 20 , 17 ,22, 23, 18 find the standard deviation.

8- If the daily expense in ten pounds of two samples of families in two adjacent cities are as follows

The first sample 24 , 20 , 26 , 25 , 30 , 18 , 32

The second sample 27 , 25 , 25 , 23 , 24 , 26

Mention with reason which of the two samples is more dispersion ?

9- The following table shows the marks of a student in mathematic within the school year

Month	Oct.	Nov.	Dec.	Feb.	March	April
Mark	36	40	42	38	46	44

Calculate the arithmetic mean and the standard deviation.

10- If the data of a number of persons in 50 families is as follows

Number of persons	2	3	4	5	6	7	8
Number of families	5	7	8	12	9	5	4

Find each of the arithmetic mean and the standard deviation of the number of persons in the families.

11- Calculate each of the arithmetic mean and the standard deviation of the following data.

The set	0-	2-	4-	6-	8-
The frequency	5	9	15	15	6

12- In a study for knowing the quantity of benzin consumed by a set of cars, the results were as follows

Number of km/litre	25-	27-	29-	31-	33-
Number of cars	5	7	9	5	4

Find the standard deviation of km/gallon

13- The following table represents a sample of health establishments due to the number of doctors in them

Number of doctors	10-	15-	20-	25-	30-	35-	40-	45 - 50
Number of health establishments	4	4	6	10	6	5	4	1

Find the standard deviation.

Models of tests in algebra and statis tics

Model (1)

1 - Complete the following :

1. From the methods of collecting data are .... , ...
2. If $(x + 5 , 8) = (1 , 6y + x)$ then $y =$ ....
3. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{\text{......} + 3c}{5b + \text{.......}} = \frac{a}{b}$
4. If $y \propto x$ and $y = 6$ when $x = 4$, then $\frac{y}{x} =$.........
5. If $n(X) = 5$, $n(X \times Y) = 15$ then $n(Y) =$ ....
6. If Ahmed answered 60% of the questions of a test with true answers and the number of questions which were answered incorrectly are 10 questions, then the number of all querstions of the test is ....

2 - Choose the correct answer from those given

1. The fourth proportional of the quantities 3 , 6 , 6 is ....
   (a) 3 (b) 6 (c) 9 (d) 12
2. The ratio between the area of a square shaped regoin of side length ℓ to the area of another square shaped region of side length 2ℓ is ....
   (a) 1 : 2 (b) ℓ : 4 (c) 1 : 4 (d) 4 : 1
3. If $3xy = 8$ then ....
   (a) $x \propto y$ (b) $y \propto x$ (c) $3x \propto 8y$ (d) $x \propto \frac{1}{y}$
4. The point (-3 , 4) lies in the .... quadrant .
   (a) first (b) scound (c) third (d) fourth
5. If the function f from the set X to the set Y , then the range of f $\subset$ ....
   (a) X (b) Y (c) $X \times Y$ (d) R

3 - (a) If $x = z + 8$ and z varies inversely as y and $z = 2$ when $y = 3$

Find y when $x = 3$

(b) Find the number that if we add it to each of the numbers 1 ,7 , 25 , then they become in continued proportional .

4 - (a) If $\frac{a}{4} = \frac{b}{5} = \frac{c}{3}$, then prove that $\frac{a - b + c}{a + b - c} = \frac{1}{3}$

(b) If X = { 2 , 3 , 4 , 7 } , Y = { 1 , 2 , 3 , 4 , 7 , 8 } , R is a relation from the set X to the set Y such that aRb means "a + b is not a prime number" for all $a \in X$, $b \in Y$. Write R and represent it by an arrow diagram.

5 - (a) If X = { 2 , 3 , 4 } , Y = { 3 , 4 , 5 , 6 , 7 , 8 } and f : X → Y where f(x) = 9 - x . Find the images of the elements of X by the function f.

(b) The following table represents the number of children of 100 families in a city

Number of children	0	1	2	3	4	Total
Number of families	6	15	40	25	14	100

Calculate each of the arithmetic mean and the standard deviation.

Model (2)

1- Complete the following :

1 . The range of the set of values 8 , 5 , 10 , 6 , 14 is ....

2. If the number 6 is the positive mean proportion of the two numbers 2 and a then a = ...

3. The point (5 , -3) lies in the ...... quadrant .

4. If X = { 2 , 3 } , then X^2 = ...

5 . If $\frac{x}{5} = \frac{y}{4} = \frac{x + y}{k}$, then k = ...

6. The teacher corrected the exam papers of one of his classes in half an hour. If the teacher took one and half an hour for correcting 120 exam - papers . Then the number of students in this class is ....

2 - Choose the correct answer from those given

1 . The simplest and easiest dispersion measure is ....

(a) the range (b) the arithmetic mean

(c) the median (d) the mode

2. If 3 a = $\frac{5}{6}$ b then $\frac{a}{b}$ = ...

(a) $\frac{18}{5}$ (b) $\frac{15}{6}$ (c) $\frac{6}{15}$ (d) $\frac{5}{18}$

3. If (3 , 5) ∈ { 3 , 6 } × { x , 8 }, then x = ...

(a) 8 (b) 6 (c) 3 (d) 5

4. If X = { 5 , 6 , 7 }, then n (X^2) = ...

(a) 3 (b) 6 (c) 9 (d) 12

5. If the point (x , 7) lies on the y - axis , then 5x + 1 = ...

(a) 0 (b) 1 (c) 5 (d) 6

6. The set of images of the elements of the domain of the function is called :

(a) the rule (b) the domain

(c) the range (d) the codomain

3 - (a) If y varies as x and $y = \frac{5}{3}$ when $x = \frac{1}{6}$

Find the value of x when $y = \frac{3}{4}$

(b) Find the number which should be subtracted from each of the numbers 3 , 7 , 19 to be in continued proportion

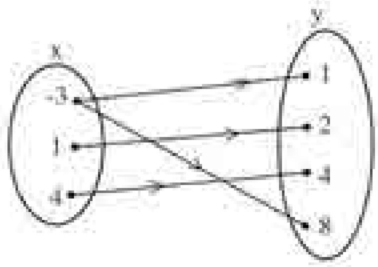
4. (a) if $= \frac{x+y}{5} = \frac{y+z}{3} = \frac{z+x}{6}$ Prove that $\frac{x-z}{2} = \frac{x+y+z}{7}$

(b) If X = { 1 , 2 } , Y = { 2 , 3 , 4 } , then find X×Y.

5. (a) The opposite arrow diagram represents the relation R from the set X to the set Y , where :

X = { -3 , 1 , 4 } , Y = { 1 , 2 , 4 , 8 },

Write R . Is R a function ? why?



Model (3)

1- Complete the following :

1. The ordered pair (x^2, y^2) where $x \neq 0$, $y \neq 0$ lies in .... quadrant
2. The positive square root of the mean of the squares of deviations of values from its arithmetic mean is called ....
3. If $\frac{a}{b} = \frac{7}{4}$ then $\frac{4a}{b}$ = ...
4. If $y^2 - 4xy + 4x^2 = 0$; then y α ....
5. If x , x + 41 are two prime numbers then x = ...
6. If X = { 2 } then X^2 = ...

2 - Choose the correct answer from those given

1. The first proportion of the quantities ...., 21 , 15 , 35

(a) $\frac{3}{7}$ (b)3 (c) 7 (d) 9

2. If $4x^2 = 9y^2$ then $\frac{x}{y}$ = ...

(a) $\frac{9}{4}$ (b) $\frac{3}{2}$ (c) $\pm\frac{2}{3}$ (d) $\pm\frac{3}{2}$

3. If $\sqrt{2}\ \sqrt{3}\ \sqrt{5}\ \sqrt{6}\ \sqrt{7}\ \sqrt{8} = 12\sqrt{70}$ then $\sqrt{3}\ \sqrt{5}\ \sqrt{6}\ \sqrt{7}\ \sqrt{8}\ \sqrt{10}$ = ...

(a) $60\sqrt{14}$ (b) $120\sqrt{7}$ (c) $12\sqrt{70}$ (c) $60\sqrt{7}$

4. If X = { 1 , 2 } , Y = { 0 } then (X x Y) = ...

(a) 0 (b) 1 (c) 2 (d) 3

5. If the point (a , 3) lies on the straight line which represents the function $f : R \rightarrow R$, where $f(x) = 4x - 5$, then a = ...

(a) 1 (b) 2 (c) 3 (d) 4

6. The set which has the greatest dispersion in the following sets is ....

(a) 28 , 17, 30 , 36 , 20 (b) 31 , 35 , 26 , 37 , 41

(c) 25 , 39 , 19 , 5 , 27 (d) 20 , 19 , 29 , 37 , 43

3 - (a) If 3 a = 2 b then find the value of $\frac{3a - b}{2a + b}$

(b) If $f(x) = x^2 - x + 3$, then find f(-2) , f (1) , f (0)

4 - (a) If y is the middle proportional between x and z

Prove that : $\frac{xz}{y(y+z)} = \frac{x}{x+y}$

(b) If y = 1 + b where b veries invesely as the square of x and y = 17 when $x = \frac{1}{2}$

Find the relation between y and X then find the value of y when x = 2

5 - (a) If R is a relation on N (set of natural numbers) , where $_aR_b$ means "a x b = 18" for all a , b ∈ N . Write R , represent it by an arrow diagram.

(b) Calculate the arithmetic mean and the standard deviation of the set of values 73 , 54 , 62 , 71, 60

(c) The ratio between two positive integers is 3 : 7. If the number 5 is subtracted from each of them, then the ratio becomes 1 : 3 find the two numbers .

Model (4)

1- complete the following

1. The arithmetic mean of the set of values 4 , 13 , 18 , 25 and 30 equals ......

2 . If $\frac{3a - 2b}{7a + 4b}$ = zero, then $\frac{b}{a}$ = ...

3. If there are 200 calory in 50 grams of one kinds of food then the number of colories in 30 grams of this food equals ....

4. If 1, x , 9 , y are in continued proortion then x = ... and , y = ...

5. The function f, where $f(x) = x^4 - 2x^3 + 7$ is polynomial of degree ....

6. If x and y are two variables and if $\frac{x_1 y_1}{x_2 y_2} = 1$ then y α ....

2 - Choose the correct answer from those given

1. If $\frac{a}{b} = \frac{5}{3}$, then $\frac{3a}{5b}$ equals ....

(a) 1 (b) $\frac{5}{3}$ (c) 3 (d) 15

2. If X = { 5 , 6 , 7 } then n (X^2) = ...

(a) 3 (b) 6 (c) 9 (d) 12

3. If $y^2 + 4x^2 = 4xy$ then ....

(a) $y \propto x$ (b) $y \propto x^2$ (c) $y \propto \frac{1}{x}$ (d) $y \propto \frac{1}{x^2}$

4. If f is an odd number then, the next odd number directly is ....

(a) f^2 (b) $f^2 + f$ (c) f + 6 (d) f + 2

5. If all the individuals are equal in values then ....

(a) $\bar{x} = 0$ (b) $\sigma = 0$ (d) $x - \bar{x} > 0$ (d) $x - \bar{x} < 0$

6. If (4 , 4) ∈ { 2 , x } × { 1 , 4 } then x = ...

(a) 2 (b) 3 (c) 4 (d) 8

3 (a) If $a^2 b^2 + \frac{1}{4} = ab$, than prove that a varies inversely as b

(b) If $\frac{a^2 + b^2}{b^2} = \frac{b^2 + c^2}{c^2}$, then prove that b is the middle proportional between a and c .

4 - (a) Find the arithmetic mean and the standard deviation of the following data

The set	0-	2-	4-	6-	8-
The frequency	5	9	15	15	6

(b) If $y \propto x$ and $y = 20$ when $x = 7$ find y when $x = 14$

5 - (a) If $X = \{1, 2, 5, 7\}$, $Y = \{2, 3, 7, 8\}$ and R is a relation from X to Y where aRb means "a + b is an odd number" for all $a \in X$, $b \in Y$. Write R and represent it by an arrow diagram. Is R a function ? why?

(b) If the weight of a body on the earth is (w) varies as its weight on the moon (r) If $w_1 = 182$ kg when $r_1 = 35$ kg find r_2 when $w_2 = 312$ kg.

Model (5)

1 - Complete the following

1. If 5 , y , 4 , 1 are proportional quantities then y = ...
2. If $y \propto x$ and $y = 2$ when $x = 8$ then y = ... when $x = 12$
3. The function f , where $f(x) = 2x - 6$ is represented graphically by a straight line intersects the x - axis at the point ....
4. If the arithmetic mean of a set of the values a , 5 , 8 , 7 , 6 equals 6 then a = ...
5. The positive middle proportional between $4a^2$ and $25b^4$ is .....
6. If $(2x, 4) = (8, y + 1)$ then $\sqrt{x^2 + y^2}$ = ...

2 - Choose the correct answer from those given:

1. If a , x , b , 2 x are proportional then $\frac{a}{b}$ = ...
   (a) 2 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4
2. If the point (x - 5 , 7 - x) lies in the second quadrant, then x = ...
   (a) 5 (b) 3 (c) 7 (d) 9
3. If $X = \{2\}$, $Y = \{0, 4\}$, then $n(X \times Y)$ = ...
   (a) 8 (b) 80 (c) 6 (d) 2
4. If the relation $R = \{(2, 3), (5, 1), (4, 6)\}$, then R represents a function whose range is :
   (a) {2 , 4 , 5} (b) {1 , 3 , 6} (c) N (d) Z
5. If the dispersion of a set of values equals zero then :
   (a) The difference between the individnals is small .
   (b) The difference between the individnals is great .
   (c) All the values if the individuals are equal
   (d) The arithmetic mean of these values is zero .

TEST

6. Notice the relation among the numbers in the pattern : 0.75 , $1\frac{1}{4}$, 1.75 , x , $2\frac{2}{4}$, ... then the value of x is ....

(a) 2.75 (b) 2.5 (c) 2.25 (d) 2

3 - (a) If $\frac{x}{y} = \frac{2}{5}$ what is the value of the expression $\frac{2x+y}{x+4y}$

(b) If $4x^2 + 9y^2 = 12xy$, then prove that x varies as y

4 - (a) Represent grtaphically the function $f(x) = (x-3)^2$, $x \in [0, 6]$ from the graph deduce the vertex of the curve , maximum (minimum) Value of the function

(b) If a , b , c and d are in continued proportion prove that : $\frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$

5 - (a) If X = { 2 , 3 , 4 , 7 } , Y = { 1 , 2 , 3 , 4 , 7 , 8 } and R is a relation from X to Y , where aRb means "a - b is an odd number" for all $a \in X$, $b \in Y$.

Write R and represent it by an arrow diagram

(b) If the velocity (V) of the water running through a pipe varies inversely as the square of the radius of the pipe (r) and v = 5 cm/sec. when r = 3 cm.

Find V when $r = 15\frac{3}{4}$ cm.

Problems for revision on Trigonometry

1 - Complete the following table:

The angle / Ratio	42° 12`	....	....	....
sin	....	0.3214	....	....
cas	....	....	0.5321	....
tan	....	....	....	2.0625

2 - Complete the following :

(1) 46° 36` 24`` = ... in degrees.

(2) 44. 125° = ... in degrees, minutes , seconds

(3) If $\tan\theta = 1.42$ where θ is the measure of an acute angle . Then θ = ...

4. If Sin θ = 0.63 where θ is the measure of an acute angle , then θ = ...

5. If Sin x = $\frac{1}{2}$ where x is an acute angle then m ($\angle$ x) = ...

6. If Cos $\frac{x}{2} = \frac{\sqrt{3}}{2}$ where x is an acute angle then m ($\angle$ x) = ...

7. Sin 60° + Cos 30° - Tan 60° = ...

8. Cos 60° + Sin 30° - Tan 45° = ...

9. 2 Sin 30° x Cos 60° - Tan 45° = ...

10. Sin^2 30° + Cos^2 30° = ...

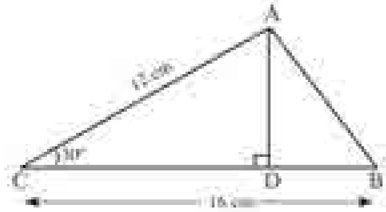
11. If tan (x + 10) = $\sqrt{3}$ where x is an acute angle then m ($\angle$ x) = ...

12. If tan 3 x = $\sqrt{3}$ where x is an acute angle then m ($\angle$ x) = ...

3 - In the opposite figure :

ABC is a triangle, $\overline{AD} \perp \overline{BC}$.

AC = 12 cm, BC = 16 cm and m ($\angle$ C) = 30°



Complete the following

$\because$ sin 30 = $\frac{AD}{\text{.........}}$

$\therefore$ AD = ... x Sin 30° = ... cm

$\because$ The area of Δ ABC = ... x AD x BC

$\therefore$ The area of Δ ABC = ... x .... x .... = ... cm^2 .

Can you calculate the height of the trianlge which is drawn from the point B on $\overleftrightarrow{AC}$?

Explain your answer showing the steps of solution .

4 - Choose the correct answer form those given :

1. 4 Cos 30° Tan 60° = ...

(a) 3 (b) 2 $\sqrt{3}$ (c) 6 (d) 12

2. If Cos 2x = $\frac{1}{2}$ where x is an acute angle then m ($\angle$ x) = ...

(a) 15° (b) 30° (c) 45° (d) 60°

3. If Tan $\frac{3x}{2}$ = 1 where x is an acute angle then m ($\angle$ x) = ...

(a) 10° (b) 30° (c) 45° (d) 60°

4. 2 Tan 45 - $\frac{1}{\text{Cos } 60}$ = ...

(a) zero (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

5. If $\cos \frac{x}{2} = \frac{\sqrt{3}}{2}$ where x is an acute angle then sin x = ...

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

6. In $\triangle$ ABC :

If m ($\angle$ A) = 85° , Sin B = Cos B,

then m ($\angle$ C) = ...

(a) 30° (b) 45° (c) 50° (d) 60°

5- Find the value of the following :

1. $(\cos 30^\circ - \cos 60^\circ)(\sin 30^\circ + \sin 60^\circ)$
2. $\frac{1}{4} \sin^2 45^\circ \tan^2 60^\circ - \frac{1}{3} \sin^2 60^\circ \tan^2 30^\circ$
3. $\sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$
4. $\dfrac{\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ}{\sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ}$

6- Prove that :

1. $\cos 60^\circ = 2\cos^2 30^\circ - 1$
2. $\tan 60^\circ (1 - \tan^2 30^\circ) = 2 \tan 30^\circ$
3. $\tan^2 60^\circ - \tan^2 45^\circ = 4 \sin 30^\circ$
4. $\tan 60^\circ = \dfrac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$
5. $\dfrac{\tan^2 30^\circ \tan 45^\circ \tan^2 60^\circ + \tan 30^\circ \tan 60^\circ}{\sin^2 60^\circ - \tan 45^\circ \sin 30^\circ} = 8$

7- Find the value of x in each of the following :

1. $x \cos 30^\circ = \tan 60^\circ$
2. $x \sin^2 45^\circ = \tan^2 60^\circ$
3. $4x = \cos^2 30^\circ \tan^2 30^\circ \tan^2 45^\circ$
4. $x \sin 30^\circ \cos^2 45^\circ = \cos^2 30^\circ$
5. $x \sin 45^\circ \cos 45^\circ \tan 60^\circ = \tan^2 45^\circ - \cos^2 60^\circ$
6. $\tan x = \dfrac{\text{Sin } 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ}{\sin 45^\circ \cos 60^\circ + \sin 45^\circ \sin 60^\circ}$

8- Find m ($\angle\theta$) where θ is an acute angle :

1. $\sin^2 45^\circ = \cos \theta \tan 30^\circ$
2. $2 \sin \theta = \tan^2 60^\circ - 2 \tan 45^\circ$
3. $\sin \theta = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$
4. $\sin \theta \sin^2 60^\circ = 3 \sin^2 45^\circ \cos^2 45^\circ \cos 60^\circ$
5. $\tan \theta = 3(\sin 30^\circ + \cos 30^\circ) - 4(\sin^3 60^\circ + \cos^3 60^\circ)$
6. $3 \tan^2 \theta = 4 \sin^2 30^\circ + 8 \cos^2 60^\circ$

9-In the Opposite Figure:

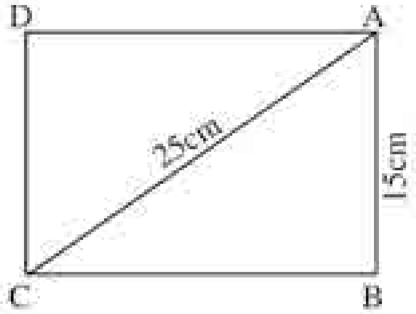
ABCD is a rectangle where AB= 15 cm

AC= 25 cm

Find:

First: m (∠ACB)

Second: The surface area of the rectangle ABCD



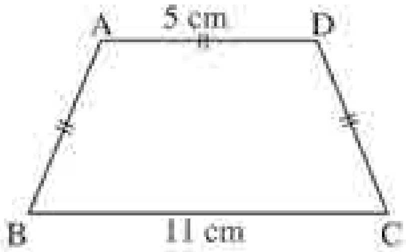
10- In the opposite figure:

ABCD is an isosceles trapezium where AB=AD=DC=5cm

BC=11cm, find

First: m(∠ B), m(∠A)

Second :the area of the trapezium ABCD.



Proplems on analytic geometry

First:Complete each of the following:

1. The distance between the two points (9,0),(4,0) is............
2. The distance between the two points (0,-11),(0,-5) is............
3. The distance between the point (4,-3) and the origin point is........
4. The distance between the two points (5,0),(0,-12) is............
5. The diameter length of the circle whose centre is (8,5)and passes through the point (4,2) equals.....................
6. If the distance between the two points (a,0) and (0,1) is one length unit then a= ....
7. The distance between the point(3,-4) and the X -axis = ... length unit.
8. In the squareABCD:If A(2,-5),B(-1,-1) then the perimeter of the square is .... lenght unit and its area is .... square unit.
9. The mid-point of the line segment joining the two points (2,5) and (4,3) is the point ....
10. If (2,1) is the mid-point of $\overline{AB}$ where A(3,-4)and B(m,6) then m= ....
11. If the Origin point is the mid-point of the line segment $\overline{AB}$ where A(5,-2) then the co-ordinates of B (..............,..............)
12. If $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB}$ = 0.75 then the slope of $\overleftrightarrow{CD}$ is ....
13. If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB}$ is 0.5 then the slope of $\overleftrightarrow{CD}$ equals.....
14. The slope of the straight line parallel to the straight line passing through the two points (2,3) and (-2,3) equals ....
15. If the straight line $\overleftrightarrow{AB}$ is parallel to X-axis where A(8,3)and B (2,K)then K =......................

16. If the straight line $\overleftrightarrow{CD}$ is parallel to the Y-axis where C(m,4)and D (-5,7) then m =................

17. ABC is a right angled triangle at B where A(1,4) and B(-1,-2) then the slope of $\overleftrightarrow{BC}$ =..................

18. If the straight line which passes through the two points (a,0) and (0,3) and the straight line which makes an angle of measure 30° with the positive direction of the X-axis are perpendicular then a=.................

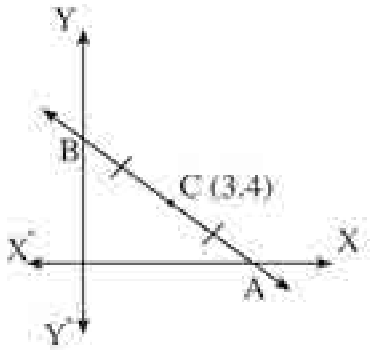
19. If $y = m\,x + c$ represents the equation of a straight line given its slope and the length of the intercepted part of the Y-axis then

 (a) The equation of the straight line when m= 1 and c=3 is ....

 (b) The equation of the straight line when m= -2 and c=1 is ....

 (c) The equation of the straight line when m= 3 and c=0 is ....

20.In the opposite figure:



C (3,4) is the mid-point of $\overline{AB}$

 (a) OA= ....length unit

 (b) OB= ....length unit

 (c) The slope of $\overleftrightarrow{AB}$ is ....

 (d) The slope of $\overleftrightarrow{CO}$ is ....

 (e) The slope of $\overleftrightarrow{AO}$ is ....

 (f) The slope of $\overleftrightarrow{BO}$ is ....

 (g) C is the centre of the circle which passes through the points ...., ..., ...

 (h) The area of Δ OAB is .... square unit.

 (i) The perimeter of Δ OAB = ... unit length.

 (j) The equation of the $\overleftrightarrow{AB}$ is ....

 (k) The equation of $\overleftrightarrow{CO}$ is ....

Second :Choose the correct answer from those given:

(1) The distance between the point (4,-3) and the X -axis equals............

 (a) -3 (b) 3 (c) 4 (d)5

(2) A cricle of centre at the origin point and its radius is 2 unit length which of the Following points belongs to the circle?

 (a) (1,2) (b) (-2,1) (c) ($\sqrt{3}$,1) (d) ($\sqrt{2}$,1)

(3) If (4,-3) is the mid -point of $\overleftrightarrow{AB}$ where A(3,-4)then the co-ordinates of B is ....

 (a) (5,-2) (b) (2,5) (c) (5,2) (d) (3.5,-3.5)

(4) The straight line whose equation is $2\chi - 3y - 6 = 0$ intercepts from the Y-axis a part of length..............

 (a) - 6 (b) - 2 (c) $\frac{2}{3}$ (d) 2

(5) If the two straight lines $3x - 4y - 3 = 0$ and $ky + 3x - 8 = 0$ are perpendicular then k=....................

(a) - 4 (b) -3 (c) 3 (d) 4

(6) If the two straight lines $x + y = 5$ and $kx + 2y = 0$ are parallel then k=..........

(a) -2 (b) -1 (c) 1 (d) 2

(7) The area of the triangle bounded by the straight lines $3x - 4y = 12$, $x = o$ and $y = 0$ in square unit equal.............

(a) 6 (b) 7 (c) 12 (d) 15

(8) $\overleftrightarrow{AB}$ is a straight line passing through the two points (2,5) and(5,2) which of the following points $\in \overleftrightarrow{AB}$

(a) (1,6) (b) (2,3) (c) (0,0) (d) (3,-4)

(9) The points (0,0) , (3,0) and (0,4)

(a) form an obtuse angled triangle (b) form an octue angled triangle

(C) form a right angled triangle (d)are collinear

(10) If A(0,0) ,B(5,7) and C (5,h) are the vertices of a right angled triangle at C then h=....................

(a) zero (b) 5 (c)7 (d)-5

Third: Answer the following questions:

(1) Find the length of $\overline{MN}$ in each of the following cases:

(a) M (2 , - 1) , N (5,3) (b) M (- 3 , - 5),N (5 , 1)

(C) M (7 , - 8) , N (2 , 4) (d) M (7 , - 3) , N (0, 4)

(2) Find the co-ordinates of the mid-point of $\overline{AB}$ in each of the following:

(1) A(2 ,4) ,B (6,0) (2) A (7 , - 5) , B (- 3 ,5)

(3) A (- 3 ,6) , B (3 , - 6) (4) A(7 ,-6) ,B(-1 ,0)

(3) If C is the mid-point of $\overline{AB}$ find x and y in each of the following cases:

First: A (1 ,5) , B (3 ,7) , C (x , y)

Second: A (- 3 , y) , B (9 , 11) , C (x , - 3)

Third: A(x ,-6) ,B(9 ,-11),C(-3 ,y)

Fourth:A(x,3) ,B(6 ,y) ,C(4 ,6)

(4) Find the slope of the straight line which makes with the positive direction of the X-axis a positive angle of measure :-

(a) $30°$ (b) $45°$ (c) $60°$

(5) Using the calculator find the measure of the positive angle which is made by the straight line whose slope is m with the positive direction of the X-axis in each of the following cases :-

(a) m = 0.3673 (b) m = 1.0246 (c) m = 3.1648

(6) Prove that the points A $(3 , - 1)$, B $(- 4 ,6)$, C $(2 , - 2)$ which belong to an orthogonal cartesian co-ordinates plane lie on the circle whose centre M $(- 1 ,2)$ then find the circumference of the circle.

(7) Find the value of a in each of the following

(a) If the distance between the two points $(a ,7)$ and $(- 2 ,3)$ equals 5.

(b) If the distance between the two points $(a , 7)$ and $(3a-1 , - 5)$ equals 13

(8) If A $(x ,3)$, B $(3 ,2)$, C $(5 , 1)$ and if AB = BC find the value of x

(9) If the points $(0 ,1)$, $(a ,3)$, $(2 ,5)$ are collinear find the value of a

(10) If the distance between the point $(x ,5)$ and the point $(6 ,1)$ equals $2\sqrt{5}$ find the value of x

(11) In which of the following cases, the points A , B and C are collinear? Explain your answer.

First: A$(-1 ,5)$,B$(0 ,-3)$,C$(2,1)$

Second: A $(- 2 , 1)$, B $(2 ,3)$, C $(4 ,4)$

Third: A $(0 ,2)$, B $(4 ,8)$, C $(6 , 11)$

(12) Identify the type of the triangle whose vertices are A $(- 2 ,4)$, B $(3 , - 1)$, C $(4 , 5)$ due to its sides lengths.

(13) Prove that triangle whose vertices A $(5 , - 5)$, B $(- 1 ,7)$, C $(15 , 15)$ is right angled at B , then calculate its area.

(14) Prove that the points $(5 ,3)$, $(6 , - 2)$, $(1 , - 1)$ $(0 ,4)$ are vertices of a rhombus, then find its area.

(15) Prove that the points A $(- 2 ,5)$, B $(3 ,3)$, C $(- 4 ,2)$ are not callinear and if D $(- 9 ,4)$ prove that the figure ABCD is a parallelogram.

(16) Let A $(5 , - 6)$, B $(3 ,7)$ and C $(1 , - 3)$. Find the equation of the straight line which passes through A and the mid-point of $\overline{BC}$.

(17) Find the equation of the straight line passing through the point $(3 , - 5)$ and parallel to the straight line $x + 2 y - 7 = 0$

(18) Find the equation of the straight line which intercepts the two axes two positive parts of lengths 4 and 9 for X and Y-axes respectively.

(19) If A (1 , - 6) , B (9 , 2) find the co-ordinates of the points which divide $\overline{AB}$ into four equal parts in length.

(20) Prove that the points A (6 , 0) , B (2 , - 4) and C (- 4 , 2) are vertices of a right angled triangle at B then find the co-ordinates of the point D which makes the figure ABCD a rectangle.

(21) If the points A (3 ,2) , B (4 , - 3) , C (- 1 , - 2) , D (- 2 ,3) are vertices of a rhombus find:

(a) The co-ordinates of the point of intersection of its two diagonals

(b) The area of the rhombus ABCD.

(22) If A (- 1 , -1) , B (2 ,3) , C (6 ,0) , D (3 , - 4) are four points on an orthogonal cartesian co-ordinates plane. Prove that $\overline{AC}$ and $\overline{BD}$ bisect each other.

What is the name of this figure?

(23) ABCD is a parallelogram where A (3 ,4) , B (2 , - 1) , C (- 4 , - 3) , find the co-ordinates of point D then find the co-ordinates of point E such that the figure ABCE becomes a trapezium in which $\overline{AE}$ // $\overline{BC}$, AE = 2 BC

(24) If the straight line L_1 passes through the two points (3 , 1) and (2 , K), and the straight line L_2 makes with the positive direction of the X -axis an angle of measure 45. Find the value of K if:

First: $L_1 // L_2$ second: $L_1 \perp L_2$

(25) Using the slope prove that the points A (- 1 ,3) , B (5 , 1) , C (6 ,4) , D (0 ,6) are vertices of a rectangle.

Model tests on Coordinate geometry and trigonometry

Model (1)

Answer the follwoing questions :

[1] Complete each of the following :

(1) If A (1,2), B (3,4), then the coordinates of the midpoint of $\overline{AB}$ is ...

(2) The equation of the straight line which is parallel to X-axis and passes through the point (-2 , 3) is ....

(3) If x,y are the measures of two complementary angles, where x :y = 1 : 2 then Sin x + Cos y = ...

(4) The distance between the points (6,0), (-4,0) equals ....

(5) If the point (0,a) belongs to the straight line 3x - 4y + 12 = 0, then a =....

(6) If $\overleftrightarrow{CD}$ // $\overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{CD} = \frac{2}{3}$. Then the slope of $\overleftrightarrow{CD}$ =.....

[2] Choose the correct answer from those given:

(1) If Cos 2 x = $\frac{1}{2}$, Then m (∠ x) = ...°

(a) 15 (b) 30 (c) 45 (d) 60

(2) The slope of the straight line whose equation 2x- 3y + 5 = 0 equals ....

(a) $-\frac{3}{2}$ (b) $-\frac{2}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

(3) The length of the line segment which is drown between the two points (0,0), (5,12) equals ....

(a) 5 (b) 7 (c) 12 (d) 13

(4) Tan 45° = ...

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\frac{1}{2}$

(5) In Δ ABC, if m (∠ B) = 90°, then Sin A + Cos C = ...

(a) 2 sin A (b) 2 Sin C (c) 2 sin B (d) 2 Cos A

(6) Tan 45° Sin 30° = ...

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

(3) (a) ABC is right-angled triangle at B,

$2AB = \sqrt{3}\ AC$, find the trigonometrical ratios of $\angle C$

(b) Find the equation of the straight line passes through the point (2,-1) and parallel to the straight line $2x - y + 5 = 0$.

(4) (a) prove that : $\cos 60° = \cos^2 30 - \sin^2 30°$

(b) ABCD is a parallelogram, its diagonals intersect at E , if A (3,-1), B (6,2) , C(1,6), then find:

First : the coordinates of E,D.

Second : the length of $\overline{DE}$

(5) (a) prove that : $\tan 60° = 2 \tan 30 \div (1-\tan^2 30°)$

(b) Find the slope, intercepted part of Y.axis of the straight line whose equation $\frac{x}{2} + \frac{y}{3} = 1$

Model (2)

Answer the following questions :

(1) Complete each of the following :

(1) If the two straight lines $2x + by + 3 = 0$,

$3x - y + 2 = 0$ are perpendicular. Then b = ...

(2) If $\sin x = 0.5$, x is an acute angle , then $m(\angle x) =$ ....°

(3) The distance between the two points (5,0) , (0,-12) equals ....

(4) $\sin 60^\circ + \cos 30 - \tan 60^\circ =$ ....

(5) If the Two straight lines $kx - 2y + 3 = 0$, $6x + 3y - 5 = 0$ are parallel , then k = ...

(6) The slope of the perpendicular straight line to the line which passes through the two points (2,6), (-4,1) equals ....

(2) Choose the correct answer from those given :

(1) $2 \sin 30^\circ \cos 30^\circ =$ ....

(a) $\sin 60^\circ$ (b) $\cos 60^\circ$ (c) $\tan 60^\circ$ (d) $2 \sin 60^\circ$

(2) The points (-3,0), (0,3) (3,0) are the vertices of a triangle

(a) Scalene triangle (b) equilateral triangle (c) Obtuse -angled triangle

(d) right-angled trianrgle and isosceles

(3) The equation of the straight line which passes through the point (2,-3), parallel to X-axis is ....

(a) $x = -2$ (b) $y = -3$ (c) $x = 2$ (d) $y = 3$

(4) If the straight line whose equation $x + 3y - 6 = 0$ is perpendicular to the straight line whose equation $ax - 3y + 7 = 0$, then a = ...

(a) 2 (b) 9 (c) - 9 (d) - 2

(5) If the point (0,4) is the midpoint of the distance between the two points (-1,-1), (x , y), then the point (x,y) is ....

(a) (1,9) (b) (-1,9) (c) ($-\frac{1}{2}$, $\frac{3}{2}$) (d) (-1,3)

(6) In ΔABC , If m ($\angle$B) = 90°, AB = 3cm

BC = 4 cm , then Sin A Cos C = ...

(a) 1 (b) $\frac{9}{25}$ (c) $\frac{12}{25}$ (d) $\frac{16}{25}$

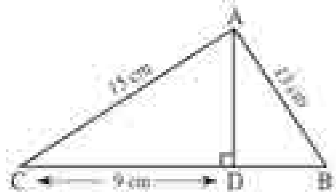
(3) (a) Find the equation of the straight line which passes through the point (1,6) and the midpoint of $\overline{AB}$, where A (1,-2), B (3, - 4)

(b) prove that : $\sin^3 30 = 9\cos^3 60 - \tan^2 45$

(4) (a) prove that the triangle whose vertices

A (1,4), B (-1,-2), C(2,-3) right at B. Then find its area.

(b) In the opposite figure :



$\overline{AD} \perp \overline{BC}$, AB = 13 cm,

AC = 15 cm , find in

the simplest form the value of : $\frac{\tan(\angle CAD) + \tan(\angle BAD)}{\tan(\angle CAD) - \tan(\angle BAD)}$

(5) (a) Find the equation of the straight line which passes through the point (3,4) and perpendicular to the straight line :

5X - 2y + 7 = 0

(b) ABCD is a trapezium in wheih $\overline{AD}$//$\overline{BC}$,

m ($\angle$B) = 90° if AB = 3cm, AD = 6cm,

BC = 10 cm , prove that :

cos ($\angle$DCB)- tan ($\angle$ACB) = $\frac{1}{2}$

TEST

Model (3)

Answer the following questions :

1- Complete each of the following :

(1) $\cos^2 45 + \tan^2 60 - \sin 30 =$ ....

(2) If A (2,-1), B (5,3), then AB = ...

(3) If $L_1 : kx - 2y + 4 = 0$,

$L_2 : x + 3y - 7 = 0$ and $L_1 \perp L_2$,

then k= ....

(4) Sin 30 cos 60 + cos 30 sin 60 = ...

(5) The equation of the straight line which passes through the point (-2,7), parallel to Y-axis is ....

(6) ΔABC right-angled at A. If tan B= 1, then tan C sin C Cos C =........

(2) Choose the Correct answer from those given :

(1) The equation of the straight line whose slope is 1 ,passes through the origin point is ....

(a) x = 1 (b) y =1 (c) y = x (d) y = -x

(1) If $\overleftrightarrow{LM} \perp \overleftrightarrow{EO}$, E (-1, 2), O (0,0), then the slope of $\overleftrightarrow{LM}$ equals ......

(a) -2 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 2

(3) If $\tan 3x = \sqrt{3}$, where (3x) is an acute, then $m(\angle x) =$ ....°

(a) 10 (b) 20 (c) 30 (d) 60

(4) If the Origin point is a centre of a circle of diameter length 6 unit length , then the point which belongs to the circle is :

(a) (6,0) (b) (0,-6) (c) $(\sqrt{8},1)$ (d) $(1,\sqrt{5})$

(5) In ΔABC, if $\angle C$ is right, then sin B+ cos B 1

(a) = (b) > (c) < (d) ≤

(6) If $\sin x = \frac{1}{2}$, x is an acute , then sin 2x = ...

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

(3) (a) Prove that the triangle whose vertices the points : Y (2,4), X (0,6.8) , Z (-5,-1) is right-angled triangle at Y.

(b) ABC is a triangle in which AB = AC = 10 cm

BC = 12 cm , $\overline{AD}$ is a perpendicular to $\overline{BC}$ intersects it at D, Prove that :

First : Sin B + Cos C = 1.4

Second : $\sin^2 C + \cos^2 C = 1$

(4) (a) Without using calculator, find the value of :

$$\frac{\cos^2 60^\circ + \cos^2 30^\circ + \tan^2 45^\circ}{\sin 60^\circ \tan 60^\circ - \sin 30^\circ}$$

(b) Represent graphically the points A (2,3), B (-1,-1), C(3,-4) , D(6,0), in the coordinate plane, then prove that they are vertices of a square, then find its area.

(5) (a) Find the value of x, where $0^\circ < x \angle 90^\circ$

if $\sin x \sin 45^\circ \cos 45^\circ \tan 60^\circ = \tan^2 45^\circ - \cos^2 60^\circ$

(b) A straight line, its slope is $\frac{1}{2}$, intersects a positive part of Y - axis of length two units, find:

First : the equation of this straight line

Second : Its intersection point with the Y - axis.

Model (4)

(1) Complete each of the following :

(1) The slope of the straight line which is parallel to the straight line which passes through the two points (3,1), (5,-1) equals ....

(2) The equation of the straight line which passes through the origin point and perpendicular to the straight line $y = 2x$ is ....

(3) The value of the expression :

sin 60 cos 30 - cos 60 sin 30 = ...

(4) If tan 3x = 1, where 3 x is an acute angle , then x = ...°

(5) The slope of the perpendicular straight line to straight line $3x + 4y - 9 = 0$ is ....

(6) If $\cos \frac{x}{3} = \frac{\sqrt{3}}{2}$, where $\frac{x}{3}$ is an acute angle, then x = ...°

(2) Choose the correct answer from those given

(1) $\text{Sin}^2 60 - \text{Cos}^2 60$ = ...

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

(2) If the origin point is a centre of a circle of radius 3 unit length, then the point .... belongs to it.

(a) (1,2) (b) $(-2,\sqrt{5})$ (c) $(\sqrt{3},1)$ (d) $(\sqrt{2},1)$

(3) The slope of the straight line which is paralld to the X-axis is ....

(a) -1 (b) 0 (c) 1 (d) undefined

(4) If the slope of straight line a x -y + 3 = 0 equals 1, then a = ...

(a) $-\frac{1}{3}$ (b) -1 (c) $\frac{1}{3}$ (d) 1

(5) The perpendicular distance between the two straight lines y-3 = 0 , y+2=0 equals ....

(a) 1 (b) 2 (c)3 (d) 5

(6) If sin 30° = cos θ , where θ is an acute angle, then m (∠θ) = ...°

(a) 60 (b) 45 (c) 10 (d) 30

(3) (a) $\overline{AB}$ is a diameter of Circle M if B(8,11), M (5,7) , then find

First : the coordinates of A

Second : the length of the radius of the circle

Third : The equation of the perpendicular straight line to $\overline{AB}$ from the point B.

(b) Find the value of x , if

$\sin x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$, where $0^\circ < x < 90^\circ$

(4) (a) ΔABC is right at C in which

AB = 10 cm , BC = 8 cm . Find the value of : Sin A Cos B + Cos A Sin B.

(b) Find the equation of the straight line which passes through the two points

(2,3) , (-3,2)

(5) (a) Without using calculator, find the numerical value of the expression:

$\cos 60^\circ \sin 30^\circ - \sin 60^\circ \cos 30^\circ$

(b) If the points A (1,0) , B (-1,4), C (7,8), D(9,4), in the coorinate plane, prove that : ABCD is a rectangle, find the length of its diagonal.

Model (5)

(1) Complete each of the following :

(1) If Sin (y + 7) = 0.5 then y = ...

(2) The equation of the straight line which passes through the point (3,-2) and parallel to X- axis is ....

(3) The distance between the point (4,3), the origin point in the coordinate plane equals ....

(4) If m_1 , m_2 are the slopes of two perpendicular straight lines, then $m_1 \times m_2$ =............

(5) 2 Sin 30 Cos 30° = Sin ....°

(6) If the straight line y = x Sin 30° + c passes through the point (4,6), then c = ...

(2) Choose the correct answer from those given:

(1) If $\frac{-2}{3}$, $\frac{k}{2}$ are the solpes of two parallel straight lines, then k = ...

(a) $-\frac{4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{1}{3}$ (d) 3

(2) If $\overline{AB}$ is a diameter of a circle, where A (3,-5) , B (5,1), then the centre of the circle is ....

(a) (4,-2) (b) (4,2) (c) (2,2) (d) (8,-2)

(3) Sin 60° + Cos 30° + Tan 60° = ...

(a) $-\sqrt{3}$ (b) $3\sqrt{3}$ (c) $\sqrt{3}$ (d) $2\sqrt{3}$

(4) If the distance between the two points (a,0), (0,1) is 1 length unit, then a = ...

(a) -1 (b) 0 (c) 1 (d) ±1

(5) The Straight line which passes through the two points (1,y) , (3,4) , its slope is tan 45° , then y = ...

(a) 1 (b) -1 (c) 2 (d) 4

(6) If Δ XYZ right at Z, XY = 25 cm,

YZ = 7 cm , XZ = 24 cm, then

Sin X + Sin Y = ...

(a) $\frac{31}{25}$ (b) $\frac{17}{25}$ (c) 2 (d) 1

(3) (a) If the two equations of two straight lines L_1 , L_2 respectively are $2x - 3y + a = 0$,

$3x + b y - 6 = 0$, then find :

First : the value of b which makes $L_1 // L_2$

Second : The value of b which makes $L_1 \perp L_2$

Third : If the Point (1,3) lies on L_1, then find the value of a.

(b) In cause of wind, the upper part of a tree was broken made an angle of measure 60° with the grourd. If the point of contact of the top of the tree with the ground was at distance 4 m from its bottom, find the length of the tree to the nearest metre

(4) (a) prove tnat : Sin 60° = 2 Sin 30° Cos 30°

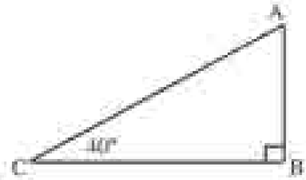
(b) ABCD is a parallelogram, A (x,2), B (3,8) , C (9,10) D (7,4) , find x

(5) (a) Prove that the triangle whose vertices A (1,-2) , B (-4,2), C (1,6) is an isosceles triangle

(b) In the oppoiste figure:

m (∠C) = 40°

AC = 12cm , find to the nearest one decimal place the length of $\overline{AB}$ and the length of $\overline{BC}$ to the nearest cm.



المواصفات الفنية:

مقاس الكتاب:	$\frac{1}{8}$ (٥٧ × ٨٢) سم
طبع المتن:	٤ لون
طبع الغلاف:	٤ لون
ورق المتن:	٨٠ جم أبيض
ورق الغلاف:	٢٠٠ جم كوشيه
عدد الصفحات بالغلاف:	١٣٦ صفحة

دار مكة المكرمة للطباعة والنشر